Density-Analog Techniques

H. C. Paxton

Los Alamos Scientific Laboratory
Los Alamos, New Mexico 74609

INITIAL FORMULATION

The original density-analog scheme was proposed in the dark ages when only the LASL subcritical "vault tests" provided guiding information about arrays of fissile material.[1] This scheme was based on the hypothesis that critical cubic arrays behave similarly to homogeneous systems when the average density of fissile material ($\bar{\rho}$) is changed by spacing variation. In other words, the critical mass of the array was represented:

$$M_c = m_{co} (\rho_0 / \bar{\rho})^s, \quad (1)$$

where m_{co} is the critical mass of a sphere (or cube) of the fissile material at full density, ρ_0, reflected like the array, and s depends upon unit size and the reflector surrounding the array. It happened that conservative interpretations of subcritical tests provided the following correlations between s and the "fraction critical" of a unit, f:[2]

$$s = 2(1 - f) \text{ for unreflected arrays,} \quad (2a)$$

$$s = 1.4(1 - f) \text{ for heavily reflected arrays.} \quad (2b)$$

The quantity f, to which we shall refer further, is defined as the ratio of the size of the fissile unit to the size of the similar critical system (same composition, density, shape, and degree of reflection as the fissile unit).

Out of Eqs. (1) and (2b) comes a relation that provides a useful rule-of-thumb for judging whether a storage array is conservatively subcritical:

$$M_c(\text{refl}) > m_{co}(\text{refl})\, \rho_0/\bar{\rho}, \text{ where } f \le 0.3. \quad (3)$$

This simplified relation is a device for the quick-and-dirty sorting of clearly safe arrangements from those that may require more detailed investigation (but, as shall be seen, it is not foolproof). We shall continue to distinguish between this limited purpose and the more ambitious function of substituting for specific criticality data.

Now that Joe Thomas has provided reliable parameters for a variety of critical reflected arrays,[3] deficiencies of the original density-analog formulation become apparent. In Fig. 1, Eq. (3) is compared with experimental, Monte Carlo, or NB_N^2 data for several families of arrays. Although most of the curves from ORNL apply to units for which f > 0.3, it is clear that Eq. (3) should not be used for large units at high storage density, and that it seriously underestimates the critical number in large arrays. In other words, this relation is satisfactory as a

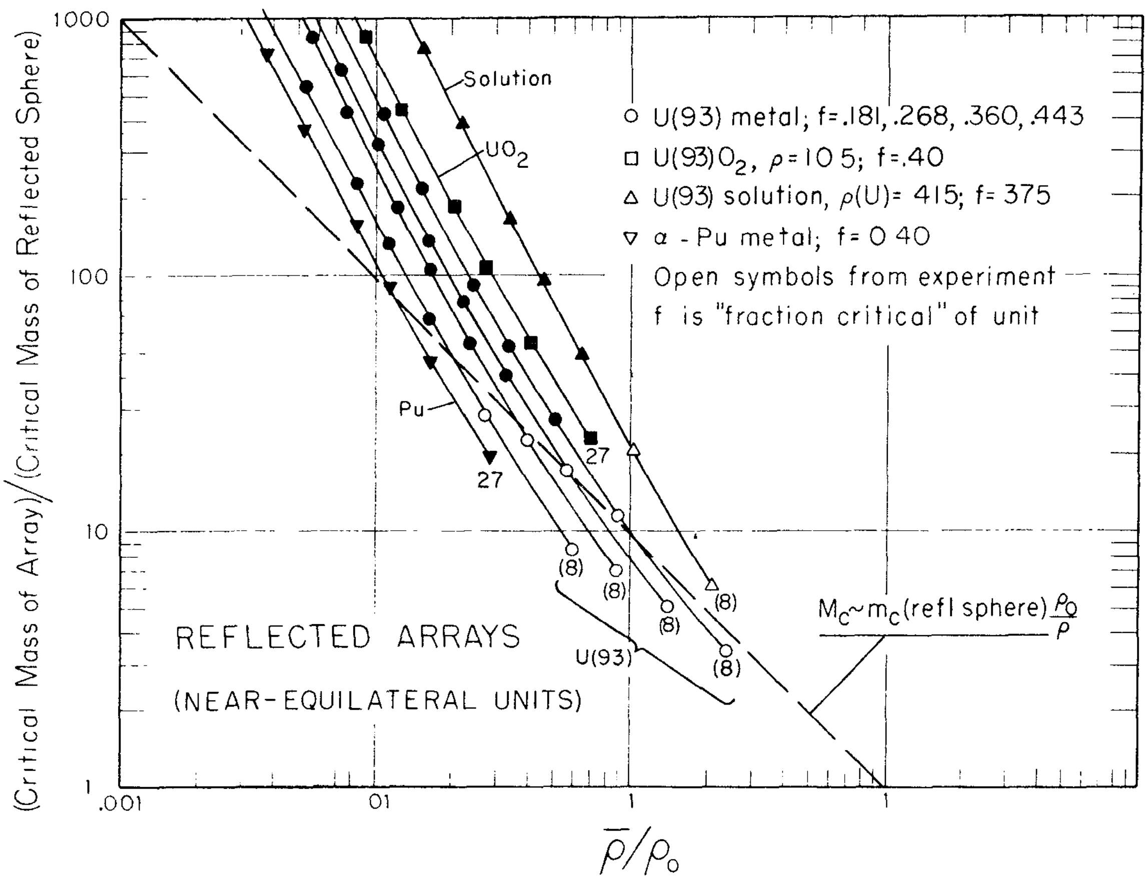

Fig. 1. Evaluation of the original density-analog formulation.

rule-of-thumb for sorting purposes if used sensibly, but it is a poor substitute for reliable data.

SMITH'S MODIFICATION

When it was demonstrated that the assumption of constant density exponent is poor for a family of reflected arrays, but better for bare arrays, a somewhat improved density-analog approximation evolved. It was assumed that results of Eqs. (1) and (2a) for bare arrays, when scaled down by a reflection factor computed for low-density homogeneous systems, would apply to reflected arrays. That is,

$$M_c(\text{refl}) = [m_{co}(\text{bare})/R]\,(\rho_0/\bar{\rho})^{2(1-f)}, \quad (4)$$

where the reflection factor R is the ratio of critical masses of corresponding bare and water-reflected spheres at very low density. Values of R computed from Hansen-Roach cross sections appear in Table I of Dave Smith's Stockholm paper.[4]

Figure 2 shows how relation (4) compares with ORNL data for reflected arrays, adjusted to apply to fissile units for which f = 0.40. [The approximation (unit mass)/ m_c (bare) $\approx$ f, for near-equilateral units, is introduced.] As suggested by the figure, this density-analog formulation is conservative for each known family of reflected arrays (even though it is now clear that homogeneous values of R are smaller than reflection factors for large arrays).

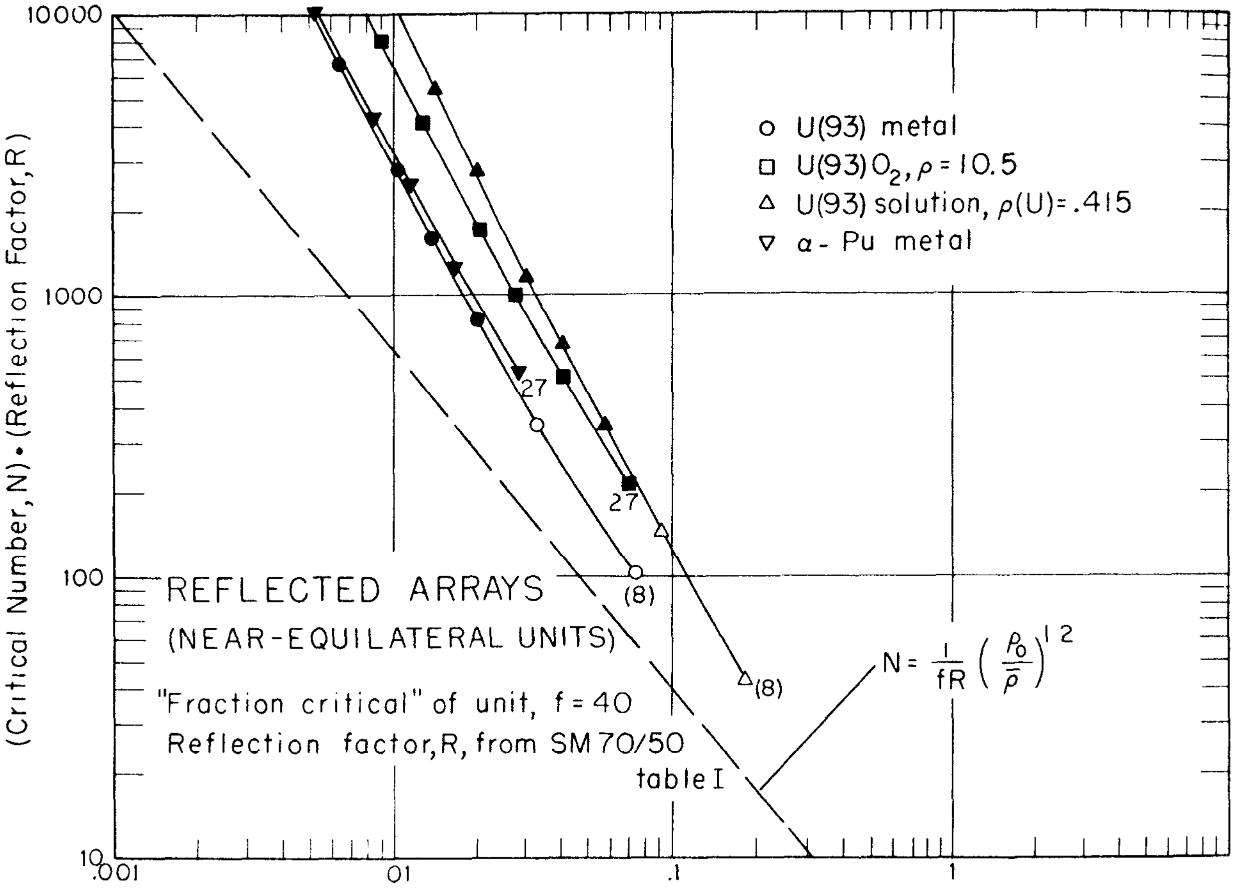

Fig. 2. Evaluation of Smith's modification.

Although the degree of conservatism is generally large, Eq. (4) has been of considerable practical value where better information was lacking. It may be noted that an arbitrary increase of the density exponent of Fig. 2 from 1.2 to 1.4 would still be conservative for all the families represented. But beyond such an adjustment little improvement is to be expected for a relation that extrapolates from a critical mass at full density, the only available starting point in the absence of data for specific critical arrays.

THOMAS' APPROXIMATION

Now that Monte Carlo array calculations have been proven, Joe Thomas suggests the usefulness of a density-analog formulation that extrapolates to smaller $\bar{\rho}$ from a computed (or experimental) critical array of reasonable size, instead of from a full-density critical mass.[3] In simple form, this version expresses the critical number of units in the array as

$$N = A\ (\rho_0/\bar{\rho})^{1.8} \text{ for large } N, \qquad (5)$$

where the constant A is evaluated in terms of the critical values of N and $\rho_0/\bar{\rho}$ for the reference array.

An alternative, illustrated in Fig. 3, is to choose the value of A so that Eq. (5) is a somewhat conservative envelope for known families of critical arrays. For this illustration, A is chosen to be suitable for f = 0.40 maximum, then adjusted by the factor 0.40/f to reduce the penalty for arrays of smaller units (f < 0.40). In this case, Eq. (5) becomes

$$N = (0.012/f)\,(\rho_0/\bar{\rho})^{1.8},\quad f \leq 0.40. \qquad (5a)$$

For certain large arrays, this expression may represent a worthwhile improvement over Eq. (4). Of course, one can foresee different choices of A and density exponent for various classes of units when array data become more abundant. This, however, implies so much reliable guiding information that a density-analog substitute for such information would hardly be necessary.

Even now, a more closely fitting value of A might be chosen for solutions (Fig. 3), but its usefulness is questionable because the storage of solution as a large number of near-equilateral units is generally impractical. As pointed out in LA-3366,[5] ORNL data on arrays of practical solution-storage cylinders can be represented

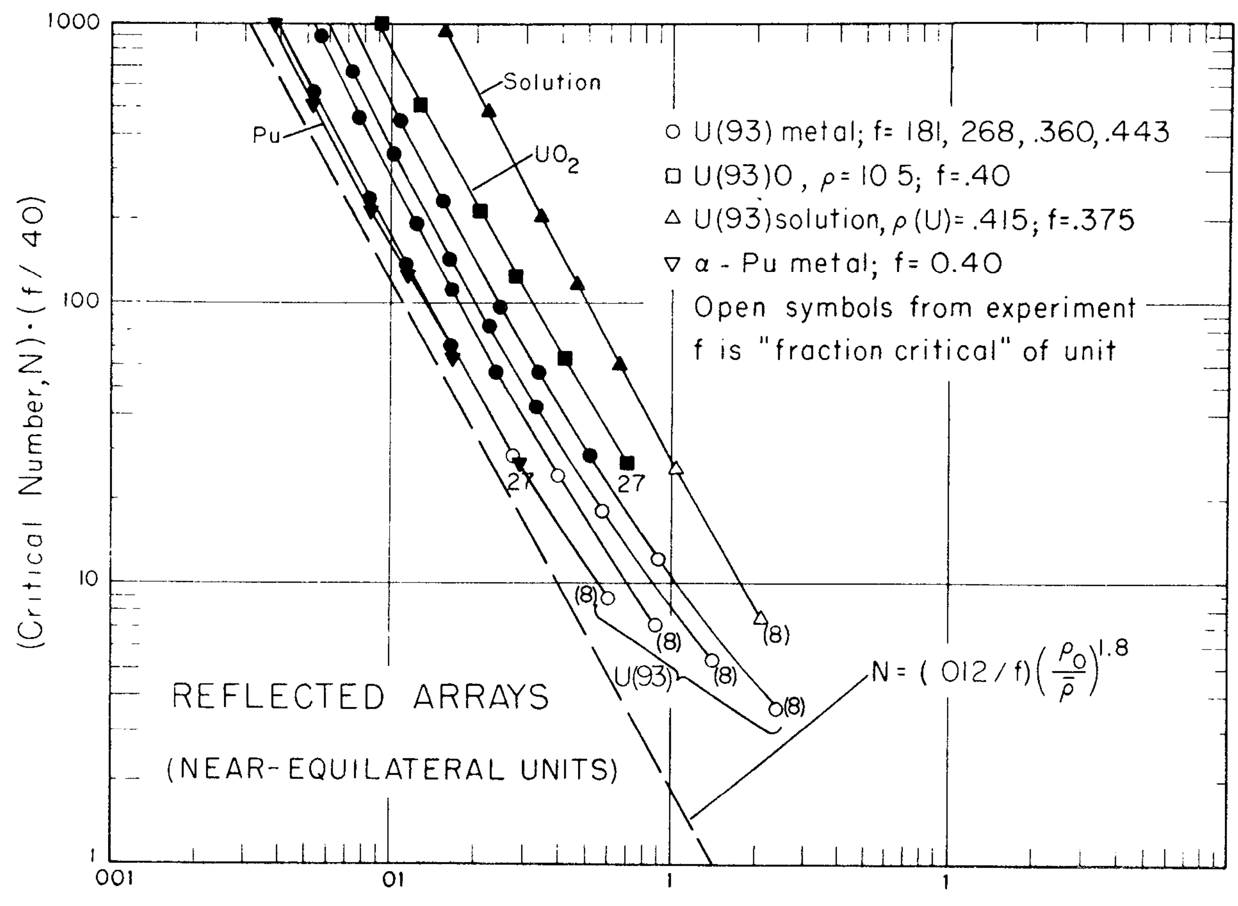

Fig. 3. Adaptation of Thomas' version.

conservatively by density-analog approximations provided a height restriction is imposed. We shall not pursue this, however, because the subcriticality of most proposed arrays of such cylinders can be judged adequately by direct comparison with the ORNL data.

SURFACE-DENSITY RULE

While touting Eq. (3) as a rule of thumb, we should not forget that some conservative limit on average surface density of stored fissile material is frequently a more useful device. The surface density ($\bar{\sigma}$) to be considered for a uniform array is the average when all fissile material is projected onto the largest face of the array. An equivalent unit is the average thickness of the fissile material ($\bar{t}$) when projected onto the largest face. In LA-3366 there is suggested a reference value of surface density for arrays of practical-sized units ($f \leq 0.3$). The plausible but unproven "limit" chosen in LA-3366 is one-half the value of $\bar{\sigma}$ or $\bar{t}$ for a critical fully-reflected infinite slab of the appropriate fissile material.

Some typical values of this limit, in half-breed units, are:

Fissile material	"Surface density" limit
U(93) solution	2 liters/ft^2
U(93) solution, $H/^{235}U \geq 500$	4 liters/ft^2
Stable $Pu(NO_3)_4$ solution	2 liters/ft^2
U(93) metal	15 kg U/ft^2
Pu metal	6.5 kg/ft^2

These tabulated values are surprisingly generous for a rule which is based on the extreme assumption of infinite extent and concomitant full reflection. For operational convenience, plant layouts are seldom so crowded as to approach these limits.

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3. H. C. Paxton, J. T. Thomas, D. Callihan, and E. B. Johnson, Critical Dimensions of Systems Containing ^{235}U, ^{239}Pu, and ^{233}U, Division of Technical Information Extension (AEC), Oak Ridge, Tennessee, Rept. TID-7028 (1964); J. T. Thomas, Criticality of Large Systems of Subcritical U(93) Components, Oak Ridge National Laboratory, Tennessee, Rept. ORNL-CDC-1 (1967); J. T. Thomas, The Effect of Unit Shape on the Criticality of Arrays, Oak Ridge National Laboratory, Tennessee, Rept. ORNL-CDC-4 (1967); J. T. Thomas, "A Method for Estimating Critical Conditions of Large Arrays of Uranium," in Nuclear Criticality Safety, Sandia Corp., Albuquerque, New Mexico, Rept. SC-DC-67-1305 (1966); J. T. Thomas, Union Carbide Corp., Oak Ridge, Tennessee, Y-12 Plant, private communication.
4. D. R. Smith, "Criteria and Evaluation for the Storage of Fissile Material in a Large and Varied Reactor Research and Development Programme," in Criticality Control of Fissile Materials (International Atomic Energy Association, Vienna, 1966).
5. H. C. Paxton, Criticality Control in Operations with Fissile Material, Los Alamos Scientific Laboratory, New Mexico, Rept. LA-3366 (1966).

DISCUSSION

E. Canfield: To calculate the average density for an array, do you put each unit in a unit cubical cell?

H. C. Paxton: Yes, that is the unique way to establish the average density.