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Reflectors, Infinite Cylinders, Intersecting Cylinders, and Nuclear Criticality

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Calculations of the effective neutron multiplication factor of critical and subcritical infinitely long cylinders of aqueous solutions of fissile materials for various configurations of water and concrete reflectors are presented These results provide a basis for investigating the criticality of intersecting pipes with similar reflectors An infinitely long central cylinder, with up to four intersections within each 0.46-m increment of length, was examined, and a method for evaluating the nuclear criticality safety of these configurations is given and a margin of subcriticality recommended

INTRODUCTION

Early nuclear criticality safety practices employed[1] the terms *minimal*, *nominal*, and *full* reflection to describe reflector conditions and to provide corresponding subcritical parameters for use in process design and in evaluation of operations with fissile materials. A minimal reflector was defined as no more than 3.2-mm-thick stainless steel or other common material such as iron, copper, aluminum, nickel, or titanium. A nominal reflector was described as one of water no more than 2.5 cm thick, or its nuclear equivalent. A full reflector was one of water at least 7.6 cm thick, or its nuclear equivalent. The use of water as a reference reflecting material stems from its effectiveness in small thicknesses, its extensive use in many critical experiments with fissile materials, and its unique specification and commonality. A recent redefinition of nominal reflection was reported[2] in which an attempt was made to clarify applications and define safe limits. The problem centers on what is meant by nuclear equivalence of the various thicknesses of water and its many interpretations in a plant environment where neutron reflection by concrete is more commonly encountered, such as in the walls and floor of a room or cell.

The effect of a neutron reflector on the neutron multiplication factor of aqueous solutions of fissile materials depends on the concentration of fissile material, the geometry, the container, and on the type and location of the reflector material. The present study utilizes the geometry of an infinite cylinder and of systems formed by intersections of cylinders of finite length with the infinite cylinder. The complex geometries of intersecting cylinders of aqueous fissile materials have received considerable attention[3-7]

[1] *Nuclear Safety Guide*, Subcommittee 8 of the ASA Sectional Committee N6, and Project 8 of the ANS Standards Committee, TID 7016, Rev. 1, U S. Atomic Energy Commission (1961).

[2] DEANNE DICKINSON, *Nucl Technol*, **26**, 265 (1975).

[3] C. L. SCHUSKE and J. W. MORFITT, "Empirical Studies of Critical Mass Data, Part II," Y-829, Carbide and Carbon Chemicals Corp (1951)

[4] VON N T. RUCKERT and W. THOMAS, *Atomkernenergie*, **21**, 197 (1973).

[5] JEAN-CLAUDE BOULY, ROBERT CAIZERGUES, EDOUARD DEILGAT, MICHEL HOUELLE, and LOUIS MAUBERT, "Interaction Neutronique dan l'Air de Recipients Cylindriques Contenant Soit des Solutions d'Uranium Soit des Solutions de Plutonium," CEA-R-3946, Service d'Etude de Criticite, Paris (1970).

[6] D. DICKINSON and C. L. SCHUSKE, *Nucl Technol*, **10**, 179 (1971).

[7] "Nuclear Criticality Safety Guide for Pipe Intersections Containing Aqueous Solutions of Enriched Uranyl Nitrate," American National Standard ANSI/ANS-8.9-1978, American Nuclear Society (1978).

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without achieving consensus of definition of magnitudes of reactivity associated with reflector conditions. Characterization of these effects is developed for specific reflector geometries of concrete and is employed as a simplified method of calculation that estimates the effective neutron multiplication factor of some simple intersections. These results have also appeared in Ref. 8.

METHODS OF CALCULATION

Many basic configurations were studied with the one-dimensional codes ANISN (Ref. 9) and XSDRN (Ref. 10), while complex geometries required the use of a Monte Carlo code. The KENO IV Monte Carlo code[11] was used to calculate problems other than the one-dimensional ones, although several of these were calculated to confirm the compatibility of results and to provide a consistent data base in making relative comparisons. Furthermore, the KENO IV code provides options that allow estimates to be made of the neutron coupling between adjacent sections of infinite cylinders and with the reflector environment. Consistent with the recommendations of Ref. 12, the codes and cross sections used have been benchmarked against experiments, and these results have appeared in publications cited below. The Hansen-Roach 16-group cross-section sets[13] were used in all the calculations performed and in the references cited.

Critical experiments with intersecting cylinders forming a "Y," a "T," and a cross were performed[14] at the Oak Ridge Critical Experiments Facility prior to 1958. The fissile solution was $U(93.2)O_2F_2$ at concentrations of 0.577 and 0.367 gU/cm^3. Criticality was achieved only in those systems closely reflected by water, i.e., submerged. Calculations of these data indicate a bias of about -0.02 in k_{eff} within one standard deviation. This bias was also observed in the calculations of critical experiments[14] with a 22.8-cm-diam stainless-steel cylinder containing solution at 0.367 gU/cm^3 and spaced at various distances from a 15.24-cm-thick slab of Oak Ridge concrete.[15] Calculations of other critical systems with materials and configurations related to this study report[16,17] a similar bias. Additional experiments of pseudo-intersecting geometries were performed[2,18] by the Critical Experiments Group at the Rocky Flats Plant in which $U(93.2)O_2(NO_3)_2$ at a concentration of 0.451 gU/cm^3 was used as the fissile material. A similar bias of -0.025 in k_{eff} was observed[19] in calculations of these systems. The most recent[20,21] critical experiments with intersecting cylinders were performed with $U(5)O_2F_2$ aqueous solutions in the concentration range from 0.745 to 0.906 gU/cm^3. Calculations of the submerged intersections accurately predict criticality.

Perhaps the most extensive application of the Hansen-Roach cross-section sets is that reported by Stratten.[22] The calculations of single units of fissile materials, reflected and unreflected, indicate that k_{eff}'s for ^{235}U, ^{233}U, and ^{239}Pu as aqueous solutions would be within the already-stated biases. Additional calculations of experiments related to this work are reported in an appendix to Ref. 15. For the fissile materials described in Table I and extensively utilized in this work, it may be expected that an overall bias of -0.02 ± 0.01 would be a reasonable and conservative value to adopt and to broadly apply to all results. The use of computed neutron multiplication factors as a substitute for experimental evidence implies that some measure of approximation to natural behavior has been accepted. Since the purpose

TABLE I

Concentrations and Atom Number Densities (atom/cm^3) of Materials Used in Calculations

	$U(100)O_2F_2$	$U(93.2)O_2F_2$	$U(93.2)O_2F_2$	$U(93.2)O_2(NO_3)_2$	$^{233}UO_2F_2$	$PuO_2 + H_2O$	$U(93.2)O_2F_2$	$U(5)O_2F_2$

[8] J T THOMAS, "Reflectors, Infinite Cylinders, Intersecting Cylinders and Nuclear Criticality," ORNL/CSD/TM-57, Oak Ridge National Laboratory (1978)

[9] W W ENGLE, Jr., "A User's Manual for ANISN, A One-Dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," K-1693, Oak Ridge Gaseous Diffusion Plant (1967).

[10] N M GREENE and C W CRAVEN, Jr, "XSDRN A Discrete Ordinates Spectral Averaging Code," ORNL-2500, Oak Ridge National Laboratory (1969)

[11] L M. PETRIE and N. F. CROSS, "KENO IV, An Improved Monte Carlo Criticality Program," ORNL-4938, Oak Ridge National Laboratory (1975)

[12] "Validation of Calculational Methods for Nuclear Criticality Safety," American National Standard ANSI/ANS-8 11, American Nuclear Society (1975)

[13] G E HANSEN and W H ROACH, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1960).

[14] J. K FOX, L. W. GILLEY, and DIXON CALLIHAN, "Critical Mass Studies, Part IX: Aqueous U^{235} Solutions," ORNL-2367, Oak Ridge National Laboratory (1958)

[15] J. T. THOMAS, "The Criticality of Cubic Arrays of Fissile Material," Y-CDC-10, Oak Ridge Y-12 Plant (1971).

[16] G. R. HANDLEY and C. M. HOPPER, "Validation Checks of the ANISN and KENO Codes by Correlation with Experimental Data," Y-1858, Oak Ridge Y-12 Plant (1972)

[17] G R. HANDLEY and C M HOPPER, "Validation of the KENO Code for Nuclear Criticality Safety Calculations of Moderated, Low-Enriched Uranium Systems," Y-1948, Oak Ridge Y-12 Plant (1974).

[18] B. B ERNST and C. L SCHUSKE, "Empirical Method for Calculating Pipe Intersections Containing Fissile Solutions," RFP-1197, Dow Chemical Company, Rocky Flats Division (1968).

[19] N F. CROSS, G. E. WHITESIDES, and R. J HINTON, *Trans. Am Nucl Soc*, **17**, 268 (1973).

[20] E B JOHNSON, *Trans Am Nucl Soc.*, **14**, 678 (1971)

[21] E. B JOHNSON, "The Nuclear Criticality of Intersecting Cylinders of Aqueous Uranyl Fluoride Solutions," Y-DR-129, Oak Ridge Y-12 Plant (1974).

[22] W R STRATTON, "Criticality Data and Factors Affecting Criticality of Single Homogeneous Units," LA-3612, Los Alamos Scientific Laboratory (1967)

TABLE I

Concentrations and Atom Number Densities (atom/cm^3) of Materials Used in Calculations

	$U(100)O_2F_2$	$U(93.2)O_2F_2$	$U(93.2)O_2F_2$	$U(93.2)O_2(NO_3)_2$	$^{233}UO_2F_2$	$PuO_2 + H_2O$	$U(93.2)O_2F_2$	$U(5)O_2F_2$
gU/cm^3 or gPu/cm^3	0.485	0.892	0.058	0.451	0.483	0.303	0.485	0.906
H:U or H:Pu	50	26.9	447	47.6	60	85	50	24.7
^{235}U	1.24284-3[a]	2.13049-3	1.38519-4	1.07686-3			1.15833-3	1.1752-4
^{238}U		1.53380-4	1.0107-5	7.7690-5			8.34477-5	2.1771-3
^{233}U					1.05206-3			
^{239}Pu						7.63590-4		
^{1}H	6.21420-2	6.14361-2	6.64891-2	5.050057-2	6.31235-2	6.49047-2	6.20886-2	5.6899-2
^{14}N				2.74290-3				
^{16}O	3.3566-2	3.52858-2	3.35418-2	3.78239-2	3.36659-2	3.39798-2	3.352787-2	3.3038-2
^{19}F	2.48568-3	4.56774-3	2.97251-4				2.48355-3	4.5892-3

[a]Read as 1.24284×10^{-3}

of the information developed is the reliable specification of subcritical configurations, there should be required an additional arbitrarily imposed margin of subcriticality in application of the information.

INFINITE CYLINDERS

The neutron multiplication factor of unreflected infinite cylinders was calculated as a function of the cylinder radius. The data are presented in Fig. 1 as a function of the fraction of the critical radius for the materials listed in the legend.

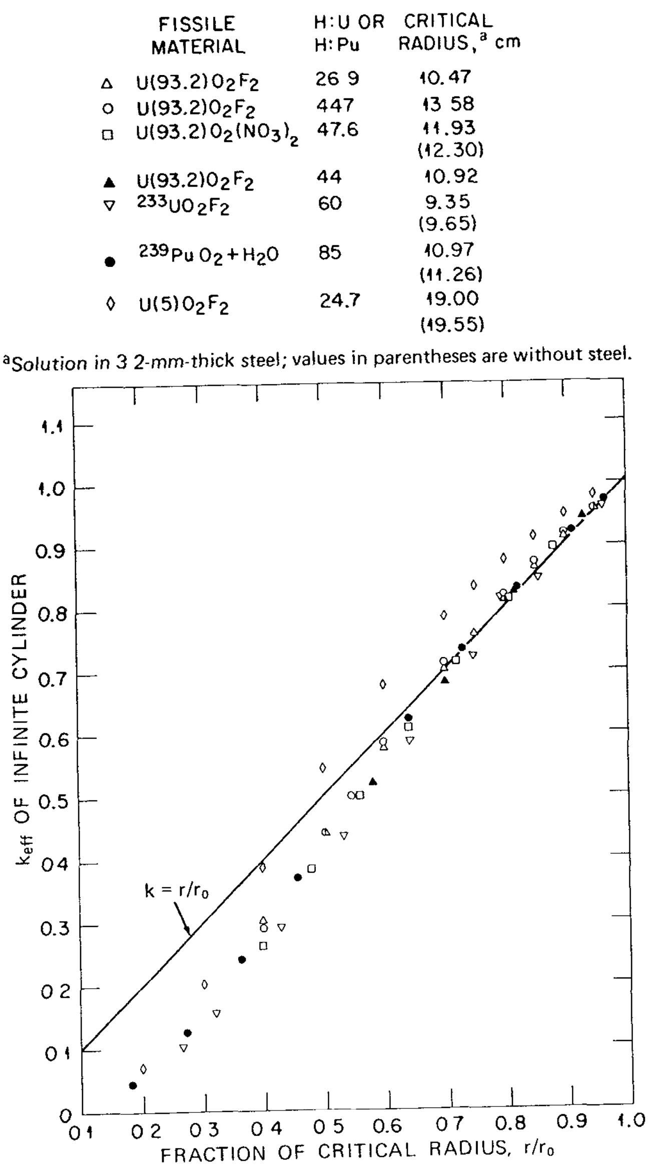


Fig. 1 The neutron multiplication factor of unreflected infinite cylinders as a function of radius for various fissile materials

A number of the cases were calculated with and without a 3.2-mm-thick carbon steel container. Most of the fissile material concentrations are in a range embracing the minimum critical dimensions for unreflected geometries. The different fissile materials, the variation in solution concentrations, and the different cylinder radii show little dispersion in this representation of the data. Each material responds similarly to the parameter r/r_0 with the exception of $U(5)O_2F_2$, where the neutron chain is being carried by a larger fraction of thermal neutrons (~0.9) than in the other materials (typically 0.3 to 0.5).

The effect on the radius of an unreflected, critical infinite cylinder as the reflector is added in increasing thickness was calculated using Oak Ridge concrete[15] (~2.3 g/cm^3) as the reflecting material. The data are shown in Fig. 2a, where the fraction of the unreflected critical radius is shown as a function of the thickness of a closely fitting reflector. Similar data of calculated critical cylinder radii from Ref. 2 for water and for concrete (~2.2 g/cm^3) as reflectors are also given for comparison.[23] The effect of a reflector on the fractional radius is different for different concentrations of fissile materials. The Δk_{eff} contribution of a reflector of given thickness to the criticality of an infinite cylinder can be estimated using Figs. 2a and 1, i.e., the reflector savings inherent in the r/r_0 value of Fig. 2a can be expressed as a Δk_{eff} by Fig. 1. For example, the r/r_0 corresponding to a water reflector thickness of 1.2 cm results in a Δk_{eff} of ~0.09, a 2.5-cm-thick reflector to a Δk_{eff} of ~0.15, and full reflection to ~0.38. These values would correspond to the loss in reactivity were the reflector removed from the cylinder. The values for the same thicknesses of concrete are ~0.05, 0.10, and 0.42, respectively. Although the steel in these configurations acts as a neutron absorber, the magnitudes of Δk_{eff} would be about the same without the steel. The fraction reduction in the unreflected infinite cylinder radius can be applied to subcritical radii, as the data in Fig. 2b for $k_{eff} < 1$ show.

The amount of reactivity to associate with a nominal reflector condition is clearly sensitive to the thickness of water defining nominal. Several reflector conditions of infinite cylinders were calculated for $U(100)O_2F_2$ aqueous solution as a function of concentration. The critical radii of an unreflected cylinder and of cylinders with

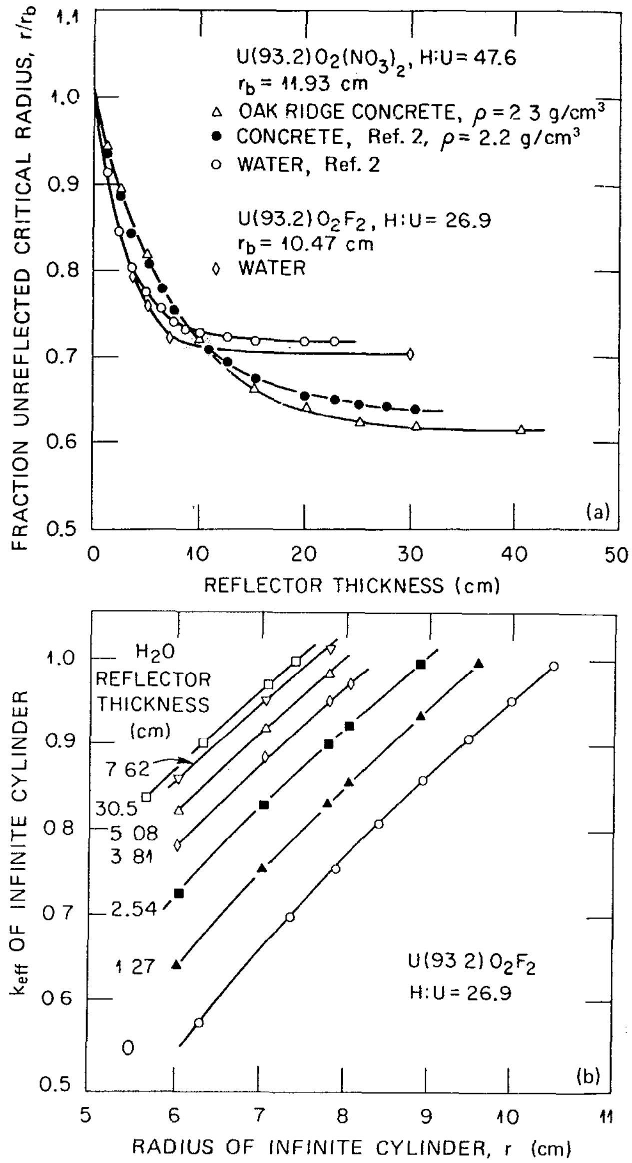


Fig 2 (a) Comparison of three different neutron reflecting materials on the critical radius of infinite cylinders as the reflector thickness increases (b) The comparison of various water reflector thicknesses on subcritical radii of infinite cylinders. A 3 2-mm-thick carbon steel shell is present as a solution container

successive additions of 3.2-mm-thick carbon steel, 2.5-cm-thick water, and 28-cm-thick water as reflectors were determined. These data are presented in Table II. The final column in Table II presents a calculation that determined the separation between surfaces of the 3.2-mm-thick steel container and the inside of a 0.4-m-thick annulus of concrete necessary to maintain criticality. The radius of the contained solution is that of the preceding column, i.e., with the 30.5-cm-thick water reflector. Water is not present in this

[23]Similar results of critical experiments with finite geometries can be examined in Ref. 24.

[24]H. C. PAXTON, J T. THOMAS, DIXON CALLIHAN, and E. B. JOHNSON, "Critical Dimensions of Systems Containing U[235], Pu[239] and U[238]," TID-7028, U S Atomic Energy Commission (1964).

TABLE II

Comparison of Critical Cylinder Radii as a Function of Reflector Condition and Fissile Material Concentration

$U(100)O_2F_2$ Solution		Reflector Condition				Separation of Inner Surface of Concrete Annulus and Solution Container[a] (cm)
		Unreflected	3.2-mm-thick Steel	3.2-mm-thick Steel 2.5-cm-thick H_2O	3.2-mm-thick Steel 30.5-cm-thick H_2O	
gU/cm^3	H:U	Critical Cylinder Radius (cm)				
1.346	15	11.797	11.503	9.713	8.033	9.599
1.096	20	11.338	11.047	9.322	7.761	9.027
0.485	50	11.052	10.744	9.054	7.677	8.175
0.1296	200	11.704	11.402	9.829	8.715	8.717
0.0524	500	13.894	13.605	12.109	11.135	11.131
0.0263	1000	18.729	18.450	17.009	16.124	16.713
0.0132	2000	55.436	55.105	53.765	53.029	53.000

[a]The water reflector has been removed from the cylinder described in the preceding column. The concrete annulus is 0.4 m thick.

configuration. These calculations define the position of the concrete as a reflector equivalent to the 30.5-cm-thick water.

Solutions having concentrations less then ~0.13 gU/cm^3 appear to require a separation about equal to the radius of the cylinder to correspond to the condition of a thick, closely fitting water reflector. A slightly greater separation is necessary for higher concentrations. Also observable in the data is the almost constant effect on the reactivity of the cylinders caused by the addition of the 3.2-mm thickness of steel, being ~0.025 in Δk_{eff} over the concentration range from 0.05 to 1.35 gU/cm^3. The magnitude diminishes for more dilute solutions.

Neutron reflection of infinitely long cylinders of solution, enclosed in 3.2-mm-thick steel, was further studied by calculating the critical radius as a function of the radial separation of the reflector and the steel container. The reflectors examined were water and concrete annuli, 0.3 and 0.4 m thick, respectively. The critical radii are presented in Fig. 3 for aqueous solutions of U(93.2) at three concentrations. The ordinate is the fraction of the unreflected critical radius, and the abscissa is the separation, S, of the outer surface of the vessel and the inner surface of the reflecting annulus. The difference in the radial fraction due to a concrete reflector replacing one of water is about the same for the concentrated fluoride and nitrate solutions and is slightly less for dilute solutions. Replacing a closely fitting thick water reflector with one of thick concrete requires a reduction in critical radius by ~14%.

It can be expected that the data for the H:U = 26.9 material would serve as a lower limit for practical process operations. Smaller critical radii have been calculated during this study for the case of S = 0. For example, a ^{233}U-metal-water mixture at H:U = 3 gave r/r_0 of 0.59 and a ^{235}U-metal-water mixture at H:U = 1 gave 0.60 for the ratio with a closely fitting concrete reflector. These latter values are not typical of aqueous solutions and would be applicable to the case of uniform slurries of fissile materials.

Additional calculations of critical radii were performed for a composite reflector of 3.2-mm-thick steel and 2.54-cm-thick water closely fitting the solution of fissile material as the annular reflectors recede to infinity. These data for a $U(93.2)O_2F_2$ solution at H:U = 26.9 are also shown in Fig. 3, where the separation is measured between the inner water reflector and the annuli. The addition of the layer of water to the cylindrical vessel changes the characteristic leakage fraction and spectrum to those resembling UO_2F_2 solution at H:U = 447. Also suggested by the data is the apparent worth of filling the 2.54-cm void between the solution and concrete with water. The radial fraction corresponding to a 2.54-cm void in the concrete annulus data is 0.64, giving $r \approx 0.64 \times 10.47 = 6.7$ cm, while a radius of 6.71 cm was calculated to be critical with the void filled with water. In all cases shown, more than half the fractional changes in radii occurs in the initial 30-cm separation of vessel and annulus surfaces.

The effect of this reflector configuration on subcritical radii of fissile material is revealed in the data of Fig. 4a for a water annulus and Fig. 4b for a concrete annulus. The fissile

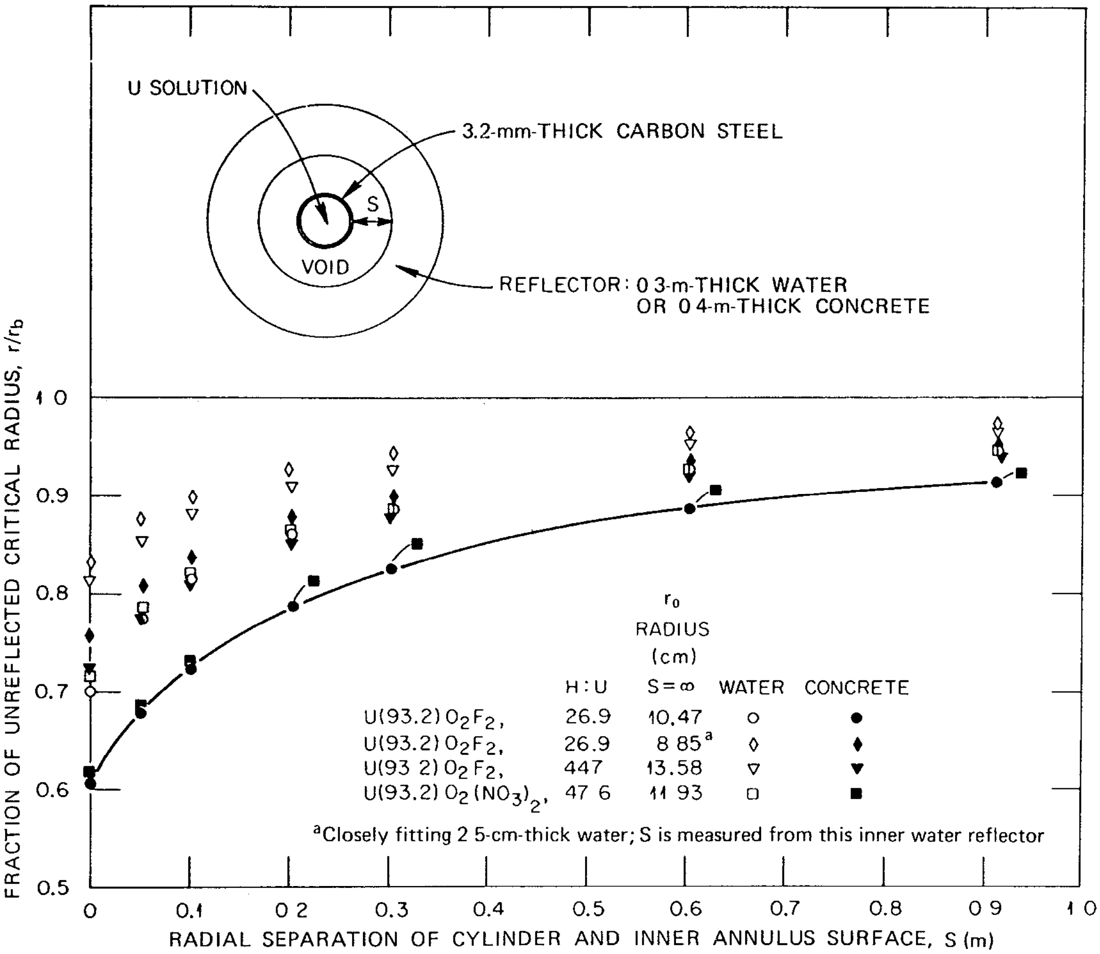


Fig. 3. The effect of annular reflectors of constant thickness on the critical radius of an infinite cylinder as the separation between cylinder and reflector increases.

material is $U(93.2)O_2F_2$ at H:U = 26.9 contained in carbon steel 3.2 mm thick. The k_{eff}'s of the reflected systems are shown as a function of the k_{eff} of the unreflected infinite cylinder and of the separation parameter, S. The data points were computed for solution radii of 8.90, 7.85, and 6.28 cm. The values shown for the k_{eff} = 1 ordinate are taken from Fig. 3 in each case. An estimate of the Δk_{eff} addition to an unreflected cylinder can be obtained from these figures for various proximate concrete or water reflectors. The magnitude represented in the figures would be conservative for many practical plant operations.

The representation of reflector effects on cylindrical geometry shown in Figs. 4a and 4b has general applicability, being useful for other fissile materials as well as composite reflectors. As an illustration of the latter, suppose a vessel has a water jacket equivalent, by calculation of infinite cylinder geometries, to the effect caused by the 3.2-mm-thick steel and 2.54-cm-thick closely fitting layer of water. The fissile material $U(93.2)O_2F_2$ at H:U = 26.9 with a radius of 7.02 cm has an unreflected k_{eff} of 0.67 from Fig. 2b, and this is effectively increased to 0.84 upon the addition of the layer of water. The latter value is shown in Fig. 5a on the $S = \infty$ line. Calculations of the r = 7.02-cm vessel with the container and layer of water within a surrounding annulus as a function of the separation parameter, S, were performed, and the data are reported in Fig. 5, where the grid of Fig. 4 has been reproduced. It is apparent that a good approximation to intermediate values can be obtained from the calculation of two points: the unreflected infinite cylinder with the layer of water only ($S = \infty$) and the infinite cylinder with a closely fitting thick reflector ($S = 0$). The two points joined by a straight line would give acceptable estimates for nuclear criticality safety purposes. This procedure persists for other subcritical radii calculated, which are identified in Fig. 5. The total Δk_{eff} controlled by reflector location is the difference of the two

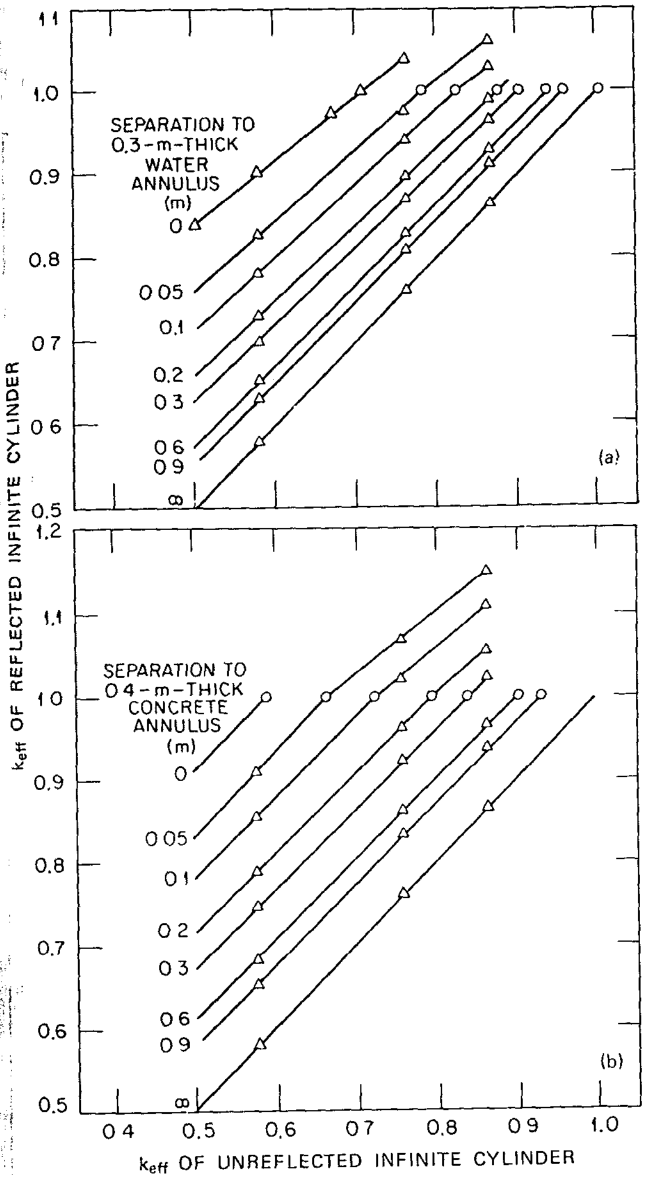


Fig. 4. The effect of a 0 3-m-thick water annulus (a) and a 0.4-m-thick concrete annulus (b) on subcritical infinite cylinders as the separation of cylinder and annulus increases The critical data represented by circles were taken from Fig 3.

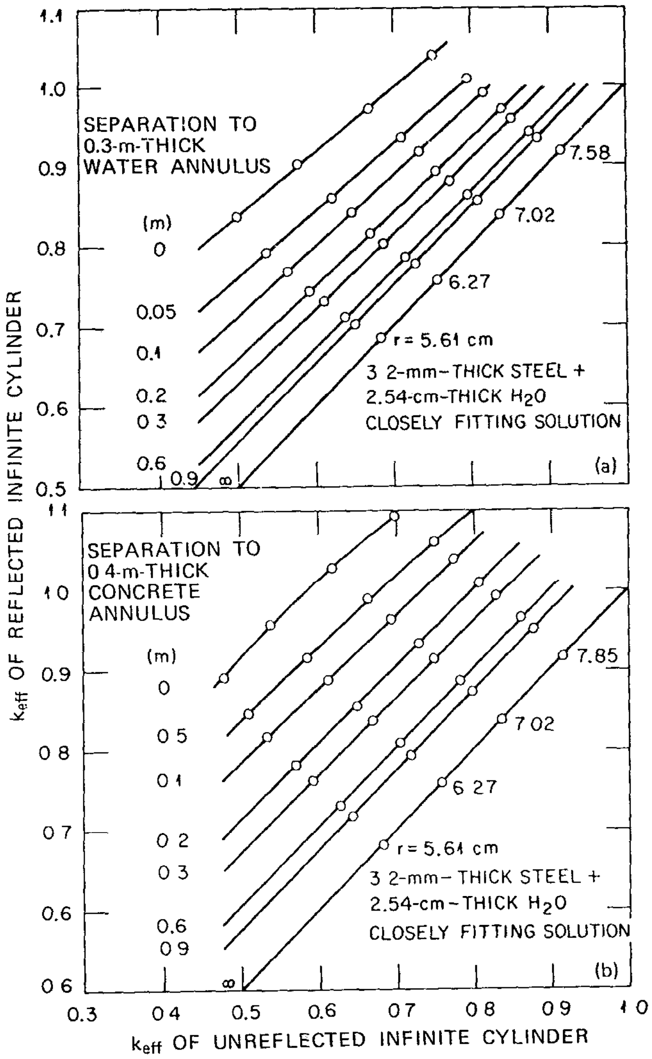


Fig. 5. The effect of a 2.54-cm-thick water reflector on subcritical radii of infinite cylinders centered in a 0.3-m-thick water annulus (a) and in a 0 4-m-thick concrete annulus (b).

extreme reflector conditions, which can be read from Fig. 2b. The lines in Figs. 4 and 5 are applicable to infinite cylinders having container materials and other thin layers of reflector materials, since these would be considered in the calculation of the configuration to define the appropriate abscissa value to be used.

INTERSECTING CYLINDERS

The effect of neutron reflector conditions on intersecting cylinders was explored for specific configurations. Generally, the k_{eff} of such intersections is dependent upon the cylinder radii, the length and relative orientation of the cylinders, the material used as a container for the solution, and the proximity of neutron reflecting materials. In describing the intersections, the larger radius cylinder is designated as the column and those of smaller radius as arms. In these calculations, the cross-sectional area of a column is divided into quadrants, each quadrant containing no more than one arm centered in the quadrant. The arms lie in a plane that is orthogonal to the column axis. The point of intersection of the plane containing the arms and the axis of the column occurs at the center of a 0.46-m length of the axis defined as a section of the column, and the sections are repeated indefinitely.

Some practical limitation to the length of the arms was necessary. Analyses of experiments have indicated that subcritical arms do not contribute significantly to the reactivity at the intersection when the arm length is more than a few diameters. The analysis of a parallel bank of long arms terminating in a single column is more properly considered as two separate problems. The intersections and the bank of arms are weakly coupled neutronically in that the interaction between the two is not significant. An example of this situation is contained in the data reported in Ref. 2. In the following calculated systems, all arms were 22.9 cm long. This limited length may result in a k_{eff} no more than 0.02 less than one would calculate for longer arms.

The systems of repeating sections were calculated to determine the k_{eff} for three different concrete reflector conditions. These are described as follows:

I. The column with intersections is centered in a square concrete annulus, 0.4 m thick, with an inside dimension of 2 m.

II. The column is positioned at the center of one side of the annulus with a surface separation between the column and the annulus equal to 30.5 cm.

III. The column is located as in condition II but with a zero separation, i.e., the column is in contact with one side of the concrete annulus.

Condition III is not applicable to intersections of four arms, i.e., when all four quadrants of the column have intersections.

The calculations were performed without a containment vessel, i.e., the column and arms are of solution only. The addition of containment materials will cause an increase in the k_{eff} of the systems. For a 3.2-mm-thick steel container, the result[25] is an increase in k_{eff} by ~0.025. Doubling this thickness would contribute an additional Δk_{eff} of ~0.02. The effect would be less if aluminum were used in place of steel.

The neutron multiplication factors for infinite cylinders, without arms, in the three reflector conditions, as well as for the unreflected cylinder, are given in Table III for the four fissile materials described therein. These data can be used with Fig. 4b to establish an equivalent reflector effect between these reflector conditions and one in which the concrete is uniformly spaced from the solution cylinder. Reflector condition I is comparable to a separation, S, of ~0.9, II to S = ~0.6, and III to S = ~0.02 m. Furthermore, the comparison reveals that for these subcritical radii, concrete appears to be most effective as a reflector for solutions of ^{233}U and least effective for solutions of ^{239}Pu.

Calculations of the repeating sections with each reflector configuration began with equal

TABLE III

The Computed Neutron Multiplication Factor of Infinite Cylinders as a Function of Reflector Condition

Cylinder Radius (cm)	Reflector Condition[a]			
	Unreflected	I	II	III
$U(93.2)O_2(NO_3)_2$ H:U = 47.6				
8.0	0.615	0.713	0.724	0.828
7.3	0.541	0.634	0.646	0.768
6.35	0.433	0.529	0.531	0.667
5.72	0.356	0.443	0.450	0.582
5.08	0.280	0.360	0.362	0.518
4.45	0.211	0.286	0.294	0.429
3.81	0.149	0.219	---	0.350
$U(100)O_2F_2$ H:U = 50				
8.0	0.704	0.796	0.809	0.918
7.3	0.619	0.711	0.714	0.841
6.35	0.504	0.592	0.610	0.740
5.72	0.427	0.506	0.521	0.650
5.08	0.343	0.433	0.425	0.577
4.45	0.262	0.336	0.347	0.494
3.81	0.185	0.253	---	0.395
$PuO_2 + H_2O$ H:Pu = 85				
7 0	0.579	0.672	0.677	0.808
6.0	0.469	0.542	0.551	0.693
5.0	0.331	0.417	0.416	0.575
4.0	0.212	0.278	0.288	0.426
3.0	0.106	---	---	---
$^{233}UO_2F_2$ H:U = 60				
7.0	0.669	0.766	0.784	0.911
6.0	0.538	0.625	0.644	0.781
5.0	0.389	0.484	0.498	0.656
4.0	0.253	0.336	0.336	0.488
3.0	0.131	---	---	---

[a]Reflector conditions: 0.4-m-thick concrete square annulus, 2-m inside dimensions:

I = column centered in annulus

II = column centered on one side of annulus, 30.5-cm surface separation

III = column centered on one side of annulus, surfaces in contact.

[25]This effect was examined in Ref 2, and similar results occur in arrays of fissile material; see Ref 26

[26]J. T THOMAS, "Some Effects of Interspersed Moderation of Array Criticality," Y-CDC-6, Oak Ridge Y-12 Plant (1969).

TABLE IV

The Computed Neutron Multiplication Factor of Repeating Sections of $U(93.2)O_2(NO_3)_2$ Solution with Zero, One, Two, Three, and Four Intersecting Arms Located in a 2-m Square, 0.4-m-Thick Concrete Annulus

Number of Arms

The Computed Neutron Multiplication Factor of Repeating Sections of $U(93.2)O_2(NO_3)_2$ Solution with Zero, One, Two, Three, and Four Intersecting Arms Located in a 2-m Square, 0.4-m-Thick Concrete Annulus

Radius[b] (cm)		Number of Arms													
		0				1			2			3			4
		Reflector Condition[a]													
Column	Arm(s)		I	II	III	I	II	III	I	II	III	I	II	III	I
		Calculated k_{eff}[c]													
8.0	8.0	0.615[d]	0.713	0.724	0.828	---	0.855	---	---	---	---	---	---	---	---
	7.3	---	---	---	---	---	0.821	---	---	---	---	---	---	---	---
	6.35	---	---	---	---	---	0.783	---	---	---	---	---	---	---	---
7.3	7.3	0.541	0.634	0.646	0.768	---	---	0.909	---	0.877	0.971	---	0.948	---	0.996
	6.35	---	---	---	---	---	---	0.845	---	0.804	0.907	---	0.853	---	0.901
	5.72	---	---	---	---	---	---	0.816	---	---	---	---	0.806	---	0.844
6.35	6.35	0.433	0.529	0.531	0.667	0.648	0.656	0.786	0.737	0.760	0.863	0.807	0.817	0.919	0.865
	5.72	---	---	---	---	0.610	0.621	0.743	0.687	0.701	0.801	---	---	---	---
	5.08	---	---	---	---	0.582	0.592	0.716	0.637	0.659	0.777	0.694	0.698	0.797	0.735
	4.45	---	---	---	---	0.558	0.569	0.696	0.607	0.615	0.733	0.638	0.644	0.740	0.673
5.72	5.72	0.356	0.443	0.450	0.582	0.567	0.583	0.717	0.657	0.671	0.779	---	---	---	---
	5.08	---	---	---	---	0.534	0.526	0.660	0.603	0.602	0.734	---	---	---	---
	4.45	---	---	---	---	0.505	0.507	0.638	0.550	0.552	0.676	---	---	---	---
	3.81	---	---	---	---	0.472	0.489	0.612	0.521	0.524	0.645	---	---	---	---
5.08	5.08	0.280	0.360	0.362	0.518	0.480	0.490	0.619	0.567	0.568	0.680	0.623	0.632	0.724	0.686
	4.45	---	---	---	---	0.434	0.442	0.579	0.507	0.510	0.631	0.558	0.571	0.658	0.589
	3.81	---	---	---	---	0.406	0.412	0.549	0.457	0.467	0.591	0.496	0.496	0.600	0.518
	3.18	---	---	---	---	0.381	0.393	0.517	0.418	0.425	0.562	---	---	---	---
4.45	4.45	0.211	0.286	0.294	0.429	0.388	0.402	0.536	0.468	0.481	0.602	0.527	0.534	0.692	0.585
	3.81	---	---	---	---	0.351	0.350	0.484	0.406	0.415	0.537	0.448	0.457	0.561	0.495
	3.18	---	---	---	---	0.328	0.325	0.453	0.365	0.370	0.490	0.393	0.400	0.501	0.420

[a]Reflector conditions: 0.4-m-thick concrete square annulus, 2-m inside dimensions:

I = column centered in annulus

II = column centered on one side of annulus, 30.5-cm surface separation

III = column centered on one side of annulus, surfaces in contact.

[b]Solutions have no containment vessel.
[c]Maximum standard deviation is ±0.007.
[d]Underlined k_{eff} values are abscissas used in plotting data in Figs. 6 through 9, e.g., all data for 5.08-cm-radius arms are plotted at k_{eff} = 0.280.

TABLE V

The Computed Neutron Multiplication Factor of Repeating Sections of $PuO_2 + H_2O$ and $^{233}UO_2F_2$ Solutions with Zero, One, Two, Three, and Four Intersecting Arms Located in a 2-m Square, 0.4-m-Thick Concrete Annulus

Radius[b] (cm)		Number of Arms: 0				1			2			3			4
		Reflector Condition[a]													
			I	II	III	I	II	III	I	II	III	I	II	III	I
Column	Arm(s)	Calculated k_{eff}[c]													
$PuO_2 + H_2O$															
7.0	7.0	0.579[d]	0.672	0.677	0.808	0.809	0.818	0.930	0.896	0.903	1.005	0.966	0.969	1.057	1.026
	6.0	---	---	---	---	0.749	0.758	0.874	0.813	0.833	0.931	0.872	0.887	0.963	0.923
	5.0	---	---	---	---	0.717	0.721	0.844	0.745	0.767	0.879	0.790	0.811	0.921	0.828
	4.0	---	---	---	---	0.689	0.693	0.829	0.724	0.728	0.845	0.736	0.745	0.845	0.748
	3.0	---	---	---	---	---	0.687	0.818	0.696	0.698	0.817	0.699	0.710	0.823	0.701
6.0	6.0	0.469	0.542	0.551	0.693	0.676	0.685	0.811	0.751	0.775	0.894	0.839	0.849	0.928	0.894
	5.0	---	---	---	---	0.609	0.619	0.758	0.673	0.685	0.809	0.728	0.740	0.844	0.763
	4.0	---	---	---	---	0.576	0.584	0.723	0.618	0.628	0.758	0.648	0.656	0.767	0.676
	3.0	---	---	---	---	0.560	0.567	0.708	0.585	0.588	0.713	0.595	0.597	0.730	0.606
5.0	5.0	0.331	0.417	0.416	0.575	0.526	0.543	0.680	0.618	0.620	0.754	0.685	0.689	0.791	0.745
	4.0	---	---	---	---	0.468	0.484	0.609	0.525	0.535	0.657	0.576	0.572	0.678	0.614
4.0	4.0	0.212	0.278	0.288	0.426	0.376	0.390	0.519	0.450	0.453	0.594	0.514	0.517	0.627	0.559
	3.0	---	---	---	---	0.319	0.324	0.461	0.366	0.368	0.502	---	---	---	0.430
3.0	3.0	0.106	---	---	---	0.227	0.234	0.292	---	---	---	0.484	0.491	0.606	0.514
$^{233}UO_2F_2$															
7.0	7.0	0.669	0.766	0.784	0.911	0.916	0.927	1.068	1.016	1.042	1.147	---	---	---	1.184
	6.0	---	---	---	---	0.867	0.865	0.995	0.937	0.951	1.075	0.993	1.009	1.096	1.044
	5.0	---	---	---	---	0.820	0.838	0.958	0.867	0.884	1.000	0.891	0.908	1.026	0.944
	4.0	---	---	---	---	0.804	0.816	0.957	0.819	0.838	0.976	0.847	0.843	0.964	0.864
	3.0	---	---	---	---	0.791	0.796	0.934	0.806	0.809	0.958	0.799	0.825	0.944	0.817
	2.0	---	---	---	---	0.782	0.787	0.908	0.782	0.793	0.931	0.786	0.788	0.937	0.794
6.0	6.0	0.538	0.625	0.644	0.781	0.778	0.787	0.949	0.858	0.879	1.010	0.935	0.949	1.073	1.009

6.0	6.0	0.538	0.625	0.644	0.781	0.778	0.787	0.949	0.858	0.879	1.010	0.935	0.949	1.073	1.009
	5.0	---	---	---	---	0.717	0.716	0.858	0.766	0.783	0.926	0.835	0.849	0.962	0.883
	4.0	---	---	---	---	0.664	0.678	0.818	0.703	0.713	0.865	0.737	0.746	0.861	0.759
	3.0	---	---	---	---	0.644	0.654	0.797	0.673	0.684	0.814	0.676	0.686	0.833	0.686
5.0	5.0	0.389	0.484	0.498	0.656	0.600	0.626	0.776	0.699	0.719	0.856	0.776	0.790	0.911	0.843
	4.0	---	---	---	---	0.537	0.547	0.705	0.599	0.611	0.766	0.649	0.656	0.778	0.694
	3.0	---	---	---	---	0.501	0.514	0.679	0.530	0.535	0.701	0.548	0.564	0.706	0.576
4.0	4.0	0.253	0.336	0.336	0.488	0.434	0.446	0.607	0.514	0.529	0.675	0.593	0.597	0.712	0.641

[a]Reflector conditions: 0.4-m-thick concrete square annulus, 2-m inside dimensions:

I = column centered in annulus

II = column centered on one side of annulus, 30.5-cm surface separation

III = column centered on one side of annulus, surfaces in contact.

[b]Solutions have no containment vessel.

[c]Maximum standard deviation is ±0.007.

[d]Underlined k_{eff} values are abscissas used in plotting data in Figs. 6 through 9, e.g., all data for 5.0-cm-radius PuO_2 solutions are plotted at k_{eff} = 0.331.

column and arm radii, and the effect of reducing the arm radius while maintaining the column radius constant was explored. In the limit, as the arm radius approaches zero, an infinite cylinder in the given reflector condition results. Data for the computed intersections are given in Table IV for the $U(93.2)O_2(NO_3)_2$ solution, and Table V presents the results for the PuO_2-water mixture and for the $^{233}UO_2F_2$ solution. The Monte Carlo calculations of Tables III, IV, and V have a maximum standard deviation of ±0.007. An examination of these data reveals that a number of the values appear to be inconsistent, i.e., larger or smaller than would be expected for a uniform variation in arm radius. This is as it should be for statistical results within one standard deviation.

Forming the difference between the k_{eff} of the intersections under different reflector conditions allows an estimate of the magnitude of Δk_{eff} associated with changing the column location. A summary of these differences in k_{eff} is given in Table VI. It should be noted that only the column and two arms are in contact with the concrete, while the third arm is normal to the concrete surface. The largest effect appears to occur for the $^{233}UO_2F_2$ solution. The average Δk_{eff} is seen to diminish with successive additions of arms. It is also evident that the variation in reactivity due to location is ~0.02 provided the system is at least 30 cm from the concrete, and this difference is independent of the number of arms and the composition of the fissile solution. However, the average change in reactivity in moving the system the remaining 30 cm to the concrete surface is 0.129 ± 0.020, the average of all 241 possible differences between condition III and condition I or II of Tables IV and V.

It is possible to extend the utility of these data and to derive more general conclusions by expressing the results analytically through empirical relations describing the results. Analyses within the 0.02 tolerance allow reflector conditions I and II to be combined. Furthermore, the data for the different fissile materials can be grouped if the k_{eff} of an unreflected cylinder is used as a correlating parameter. Finally, we impose a constraint that only data for equal column and arm radii are considered.

These conditions permit a least-squares fit of the limited data base to the linear relation,

$$k_{eff}(R) = a_0 + a_1 k_{eff}(u) \quad , \qquad (1)$$

where the parameter R specifies reflector condition I, II, or III and (u) designates the unreflected cylinder condition. The determined coefficients a_0 and a_1 are summarized in Table VII for the reflector conditions and the number of arms, n,

TABLE VI

Difference in Δk_{eff} Values Between Reflector Condition III and Reflector Conditions I and II for ^{235}U, ^{233}U, and ^{239}Pu

	k_{eff}(III) - k_{eff}(I)				k_{eff}(III) - k_{eff}(II)				Overall Average
	1 Arm	2 Arms	3 Arms	Average	1 Arm	2 Arms	3 Arms	Average	
U(93.2)$O_2(NO_3)_2$, H:U = 47.6									
Δk_{eff}	0.1374	0.1276	0.1120	0.1278	0.1299	0.1164	0.1048	0.1188	0.1232
$\pm\sigma$	0.0072	0.0084	0.0204	0.0151	0.0051	0.0115	0.0208	0.0155	0.0159
$^{239}PuO_2 + H_2O$, H:Pu = 85									
Δk_{eff}	0.1336	0.1314	0.1114	0.1255	0.1250	0.1207	0.1036	0.1167	0.1210
$\pm\sigma$	0.0229	0.0099	0.0151	0.0193	0.0212	0.0119	0.0154	0.0188	0.0194
$^{233}UO_2F_2$, H:U = 60									
Δk_{eff}	0.1539	0.1522	0.1337	0.1469	0.1437	0.1379	0.1225	0.1350	0.1410
$\pm\sigma$	0.0174	0.0124	0.0163	0.0176	0.0143	0.0161	0.0163	0.0176	0.0185

TABLE VII

Coefficients a_0 and a_1 of Eq. (1) from the Data of Tables IV and V for Equal Column and Arm Radii

Reflector Condition	Number of Arms	Coefficients a_0	a_1	$\pm\sigma$ (k_{eff})	Equation Number[a]
I, II	0	0.063	1.058	0.013	(1a)
III	0	0.223	1.022	0.015	(1b)
I, II	1	0.153	1.151	0.011	(1c)
III	1	0.302	1.137	0.020	(1d)
I, II	2	0.221	1.201	0.016	(1e)
III	2	0.368	1.149	0.019	(1f)
I, II	3	0.254	1.258	0.029	(1g)
III	3	0.374	1.247	0.027	(1h)
I	4	0.302	1.303	0.019	(1i)

[a]Substitution of the respective pairs of coefficients in Eq. (1) results in the formation of Eqs. (1a) through (1i).

in the intersection. Substitution of the respective pairs of coefficients given in Table VII into Eq. (1) results in the formation of Eqs. (1a) through (1i). The standard deviation of the k_{eff} calculated from Eqs. (1a) through (1i) is also given.

The equations give Δk_{eff} results comparable to those of Table VI. For example, Eqs. (1c) and (1d) for one-arm intersections give $\Delta k_{eff} = 0.135 \pm 0.019$ at $k_{eff}(u) = 1$ and 0.138 ± 0.019 at $k_{eff}(u) = 0.8$ for the difference between reflector condition I or II and condition III. Similar results can be obtained by taking the difference of pairs of equations in Table VII for equal n.

The data of Tables IV and V are presented in Figs. 6 through 9 where the k_{eff} of the repeating intersections for the specified reflector condition is shown as a function of the k_{eff} for an unreflected infinite cylinder with a radius equal to that of the arm. The straight lines are defined by the relations of Table VII, and the two light lines define the standard deviation. The darkened symbols are the data for equal column and arm radii. The data branching from the line toward the ordinate are those of reduced arm radii with the column radius at the constant initial value. The ordinate values are, of course, those of an infinite cylinder having the column radius and the reflector condition of the intersection.

INTERSECTIONS IMMERSED IN WATER

Calculations of single 0.46-m-long sections with the intersections at the midpoint were performed with a closely fitting water reflector of an effectively infinite thickness. These data are presented in Table VIII. Except for the entries in one column of the U(93.2)$O_2(NO_3)_2$ data, there is no container material between the solutions and water. Unlike the previous results, the introduction of carbon steel as a container does lower the calculated k_{eff} values. There are sufficient data to interpolate radii appropriate to a margin of subcriticality corresponding to a k_{eff} of 0.9 for practical applications. The dimensions of the U(100) and U(93.2) intersections are proportional to their unreflected critical radii,

Fig. 6. The assembly of rep one intersecting 0.4-m-thick con from concrete s tered on one si as the arm radiu

as previousl of a three-ar carbon steel were perfor by concrete. nitrate solu entry of Tab 0.014, and radius gave of 0.07 in th results for ence of the c

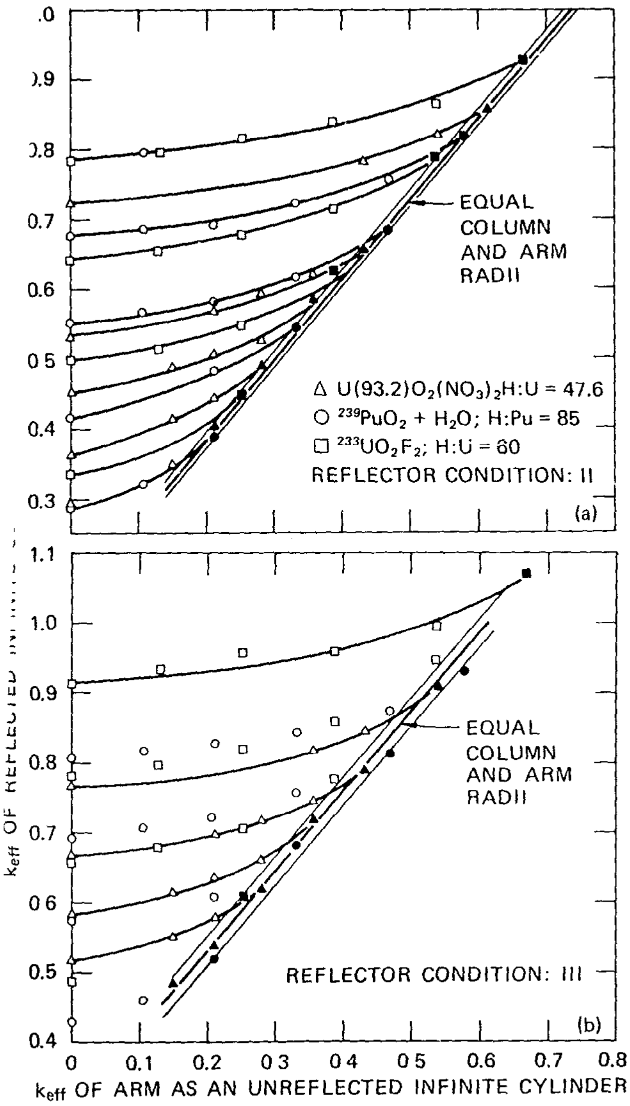


Fig. 6. The neutron multiplication factor of an infinite assembly of repeating 0 46-m-long sections, each comprised of one intersecting arm The assembly is located in a 2-m square, 0 4-m-thick concrete annulus; (a) assembly at least 30 cm distant from concrete surfaces; (b) assembly in contact with and centered on one side of annulus Column radius remains constant as the arm radius varies from the column dimension to zero.

as previously noted. Two additional calculations of a three-arm intersection with the 3.2-mm-thick carbon steel container and the U(93.2) solution were performed with the section closely reflected by concrete. The three-arm intersection of uranyl nitrate solution with a radius of 5.70 cm, an entry of Table VIII, resulted in a k_{eff} of 0.939 ± 0.014, and a second calculation with a 4.56-cm radius gave a value of 0.793 ± 0.011. The Δk_{eff} of 0.07 in the first case is comparable to previous results for concrete replacing water if the presence of the carbon steel is considered.

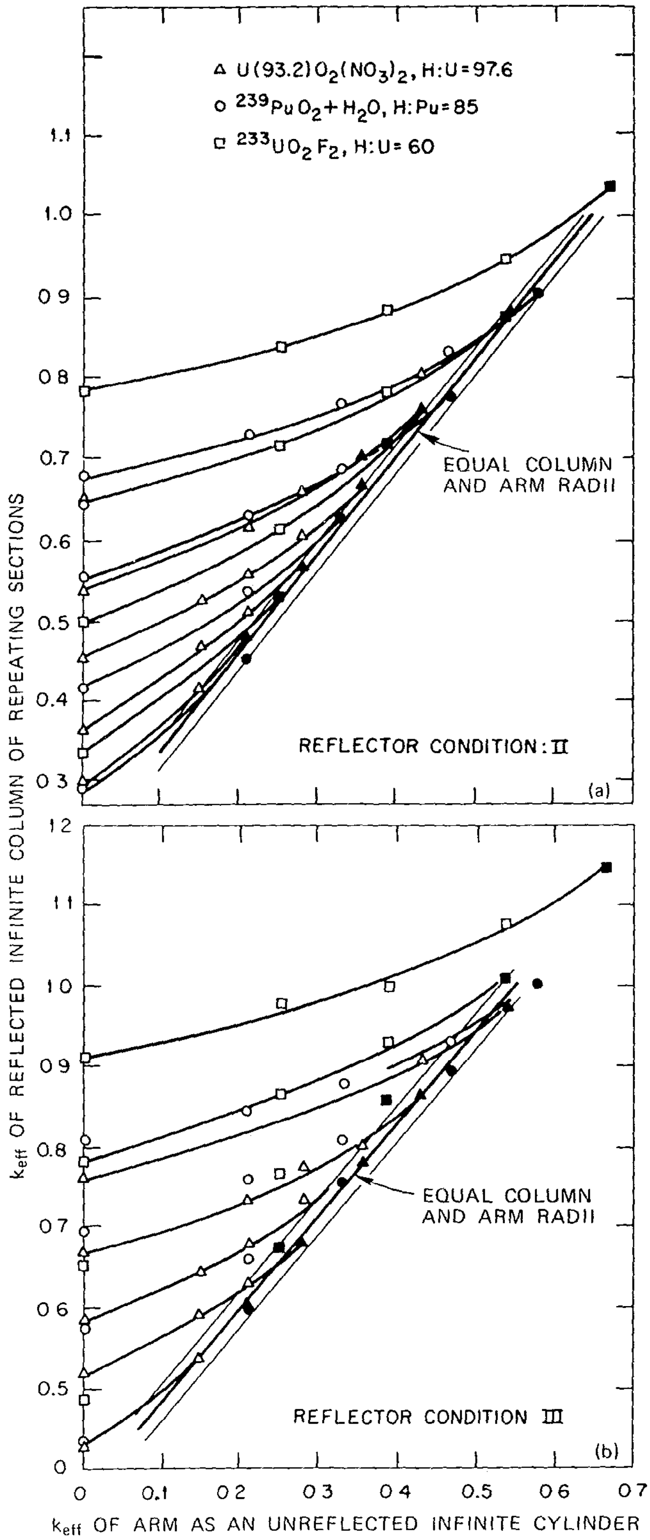


Fig 7 The neutron multiplication factor of an infinite assembly of repeating 0 46-m-long sections each comprised of two intersecting arms The assembly is located in a 2-m square, 0 4-m-thick concrete annulus; (a) assembly at least 30 cm distant from concrete surfaces; (b) assembly in contact with and centered on one side of annulus. Column radius remains constant as the arm radius varies from the column dimension to zero

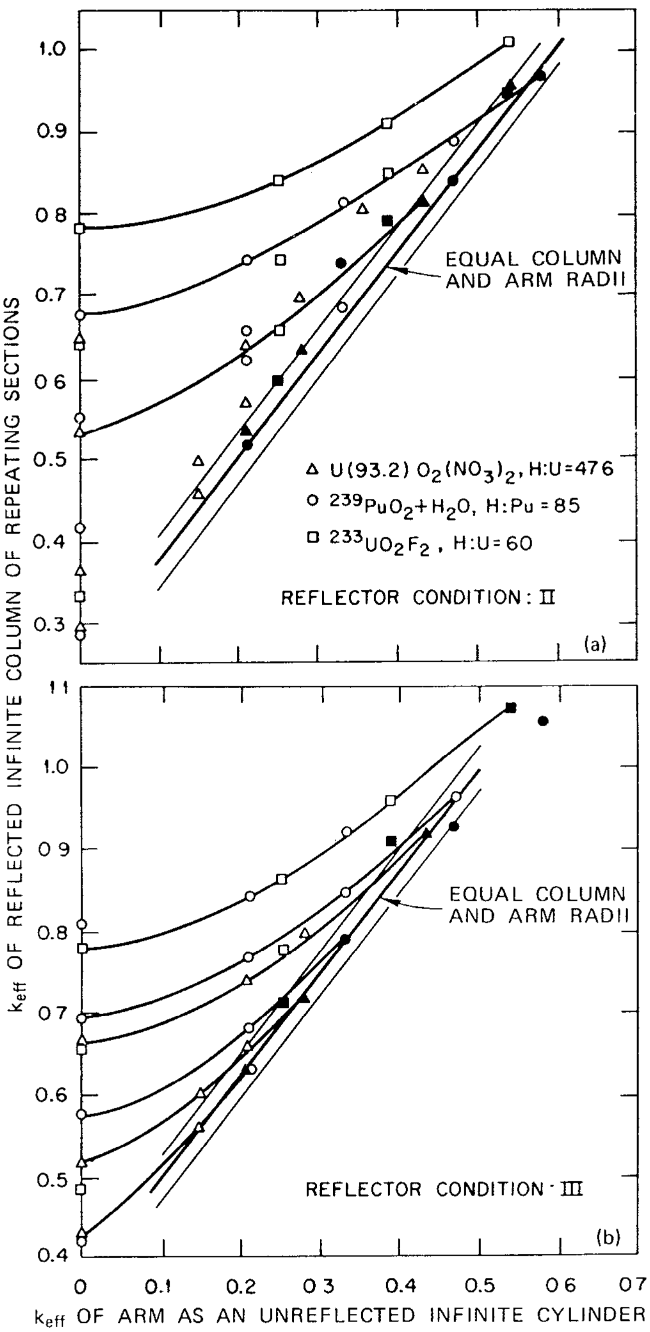


Fig. 8 The neutron multiplication factor of an infinite assembly of repeating 0.46-m-long sections, each comprised of three intersecting arms The assembly is located in a 2-m square, 0 4-m-thick concrete annulus; (a) assembly at least 30 cm distant from concrete surfaces, (b) assembly in contact with and centered on one side of annulus. Column radius remains constant as the arm radius varies from the column dimension to zero

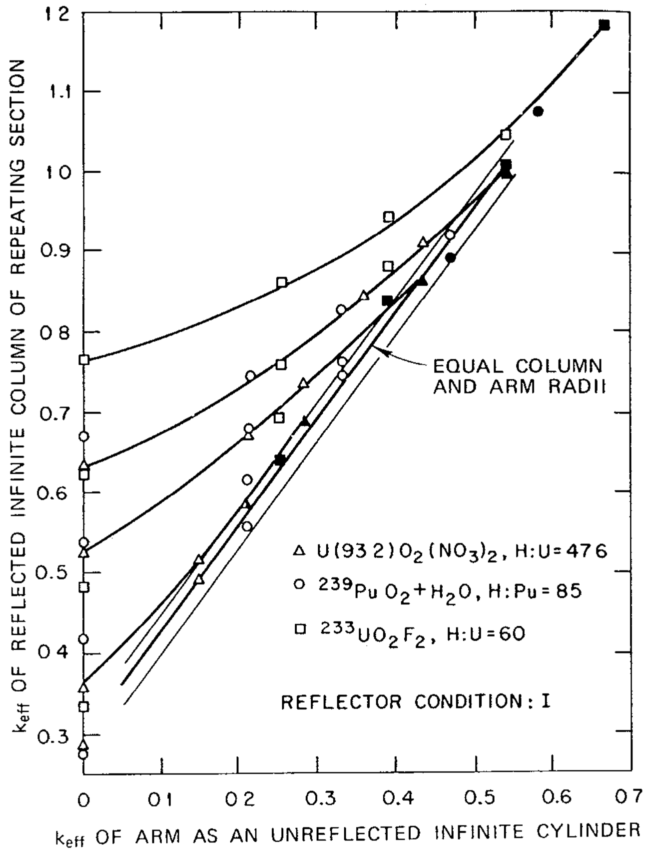


Fig. 9. The neutron multiplication factor of an infinite assembly of repeating 0.46-m-long sections, each comprised of four intersecting arms. The assembly is centered in a 2-m square, 0.4-m-thick concrete annulus. Column radius remains constant as the arm radius varies from the column dimension to zero

The four-arm intersection of $^{233}UO_2F_2$ with r = 3.56 of Table VIII was repeated indefinitely and was calculated to have a k_{eff} of 0.890 ± 0.010, indicative of neutron decoupling of sections in water. Although the arm interaction is totally suppressed in water, statistics obscure an expected increase in k_{eff} of ~0.02 due to continuation of the column of solution.

A more systematic calculation of $U(5)O_2F_2$ solution at H:U = 24.7 is given in Table IX for submerged sections. The maximum number of arms considered was two. These data also can be represented graphically as in Figs. 6 through 9. For equal column and arm radii, the k_{eff} appears rather insensitive to the number of arms and their orientation to the column, the total Δk_{eff} being less than 0.05 in these data. As further illustration, consider a submerged critical infinite cylinder. There is

1. a slight increase in reactivity if the cylinder has a 90-deg bend

2. a Δk_{eff} increase of ~0.02 if an arm of equal radius intersects the cylinder

3. a Δk_{eff} increase of 0.03 if an added arm of equal radius is inclined 30 deg to cylinder

TABLE VIII

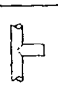
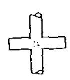
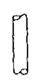
Calculated Neutron Multiplication Factors for a Submerged Intersection with One, Two, Three, and Four Arms

Number of Arms	Radius,[a] k_{eff}	$U(93.2)O_2(NO_3)_2$ H:U = 47				$PuO_2 + H_2O$ H:Pu = 85		$^{233}UO_2F_2$ H:U = 60		$U(100)O_2F_2$ H:U = 50	$U(93.2)O_2F_2$ H:U = 50
1	r (cm)	5.30	6.46	7.60	6.96[b]	4.77	6.15	3.97	5.00	6.00	6.00
	k_{eff}	0.812	0.943	0.988	0.895	0.797	0.914	0.789	0.892	0.926	0.907
2	r (cm)	5.10	5.86	7.29	6.50[b]	4.59	5.20	3.82	4.54	---	---
	k_{eff}	0.817	0.941	1.054	0.919	0.822	0.902	0.808	0.925	---	---
3	r (cm)	4.95	4.95	7.08	5.70[b]	4.46	4.60	3.71	3.70	---	---
	k_{eff}	0.898	0.889	1.089	0.870	0.887	0.890	0.892	0.842	---	---
4	r (cm)	4.6	4.75	6.79	5.45[b]	4.28	4.28	3.56	3.56	4.70	4.70
	k_{eff}	0.861	0.914	1.102	0.891	0.903	0.889	0.913	0.875	0.929	0.912

[a]Radii of column and arms are equal. Maximum σ of k_{eff} is 0.025.
[b]Solution contained in 3.2-mm-thick carbon steel.

TABLE IX

Calculated Neutron Multiplication Factors for Submerged Repeating Sections of $U(5)O_2F_2$ Intersections for Configurations Shown

Radius (cm)		Configuration of Intersection				
			 [a]			
Column	Arm	Calculated k_{eff}[b]				
15.24	15.24	1.027	1.041	1.019	1.059	---
	12.73	1.011	1.024	0.990	1.031	---
	10.14	1.002	1.011	0.990	1.002	---
	7.70	0.998	1.002	0.988	0.998	---
	0					1.008
12.73	12.73	0.947	0.979	0.908	0.972	---
	10.14	0.928	0.937	0.898	0.934	---
	7.70	0.926	0.921	0.901	0.926	---
	0					0.911
10.14	10.14	0.851	0.867	0.795	0.874	---
	7.70	0.816	0.821	0.791	0.836	---
	0					0.803
7.70	7.70	0.705	0.716	0.645	0.736	---
	0					0.676

[a]Angle between arm and column axes is 30 deg.
[b]Standard deviation $\leq$0.005.

4. an increase of Δk_{eff} = 0.04 if two arms are added.

If the infinite cylinder is subcritical submerged with $k_{eff} \approx$0.9, there is

1. no detectable gain in k_{eff} for a 90-deg bend
2. a positive Δk_{eff} of 0.04 if an arm of equal radius is added
3. a positive Δk_{eff} of ~0.07 if an arm of equal radius is inclined 30 deg to the cylinder
4. a Δk_{eff} increase of ~0.06 if two arms of equal radius form a cross with the cylinder.

The k_{eff}'s are less if the arms have radii less than the column. The addition of carbon steel as a container material will also decrease k_{eff}.

APPLICATIONS AND DISCUSSION

Equation (1) can be extended to include the dependence of k_{eff} on the number of arms, n, for reflector condition I or II. Examination of the a_0 values of Table VII shows that a_0 is augmented by ~0.05 per arm as n increases from 1 through 4. The additional observation that the coefficient a_1 can be expressed approximately as $(1 + a_0)$ results in the following relation:

$$k_{eff}(n,\mathrm{II}) = 0.05(n + 2) + [1 + 0.05(n + 2)]k_{eff}(0,u) \quad , \tag{2}$$

where n = 1, 2, 3, or 4. Comparison with corresponding equations for n in Table VII defines the maximum differences in $k_{eff}(n,\mathrm{II})$ at $k_{eff}(0,u) = 1.0$, and these, combined with the associated σ, yield an expected error in k_{eff} of $\pm$0.03 for all n. Application of Eq. (2) to $n = 0$ values, i.e., infinite cylinders, would result in conservative estimates of $k_{eff}(0,R)$.

Since the effect of changing intersections from reflector condition II to III augments the k_{eff} by ~0.13 for subcritical radii, this value can be added to the result of Eq. (2) to estimate the neutron multiplication factor of intersections against a concrete wall, $k_{eff}(n,\mathrm{III})$. Applications of these data and results to practical design problems require that an adequate margin of subcriticality be adopted for planned operations. Considering variations in chemical concentrations ($\Delta k_{eff} \approx 0.03$), the bias in calculating solution systems (~0.02), the influence of container materials (~0.05), and the requirement for a minimum margin of subcriticality (0.05), a Δk_{eff} of 0.15 is required and a limit for Eq. (2) would be $k_{eff}(n,\mathrm{II}) \leq 0.85$. Similarly, to yield $k_{eff}(n,\mathrm{III}) \leq 0.85$ would require $k_{eff}(n,\mathrm{II}) \leq 0.72$ as a limit for Eq. (2). These limits are consistent with Ref. 7 and are prudent.

These criteria allow an estimate of the dimensions of intersecting pipes applicable to plant design problems. Explicitly, this can be accomplished by rewriting Eq. (2) for k_{eff} of an infinite cylinder as

$$k_{eff}(0,u) = \frac{k_{eff}(n,\mathrm{II}) - 0.05(n+2)}{1 + 0.05(n+2)} \quad ,$$

giving, for example when $k_{eff}(n,\mathrm{II}) = 0.85$, the maximum $k_{eff}(0,u)$ equal to 0.608, 0.542, 0.480, and 0.423, corresponding to n = 1, 2, 3, and 4, respectively. These k_{eff}'s are readily expressed as dimensions through a relation typified by data in Fig. 1. Similar dimensions can be defined for $k_{eff}(n,\mathrm{III})$. The range of $k_{eff}(0,u)$ values elicits the following two remarks concerning design of pipe intersections: First, the dimensions to be considered are such that the infinite cylinder having a thick water reflector is subcritical. Second, an evident rule that may be useful in the field (away from calculators) is that $k_{eff}(0,u)$ is conservatively approximated in Eq. (2) by the relation $k_{eff} = r/r_0$, as demonstrated in Fig. 1.

The representation of data in Figs. 6 through 9 suggests a method for estimating $k_{eff}(n,\mathrm{II})$ for arms of reduced radii. The loss in $k_{eff}(n,\mathrm{II})$ due to a reduction in the arm radius is conservatively approximated by the linear relation

$$\Delta k_{eff}(n,\mathrm{II}) = \left[1 - \frac{k^a_{eff}(0,u)}{k^c_{eff}(0,u)}\right] \left[k_{eff}(n,\mathrm{II}) - k^c_{eff}(0,\mathrm{II})\right] \quad , \tag{3}$$

where the superscript a refers to the arm, c to the column, and $k^c_{eff}(0,\mathrm{II})$ is the effective neutron multiplication factor of an infinite column in the reflector condition of the intersection.

As an example, consider the three-arm intersection of $^{233}UO_2F_2$ in Table V having column and arm radii of 6.0 and 4.0 cm, respectively. The $k^c_{eff}(0,u)$ is 0.538, $k^c_{eff}(0,\mathrm{II})$ is 0.644, and $k^a_{eff}(0,u)$ is 0.253 from Table III. The $k_{eff}(n,\mathrm{II})$ from Eq. (2) is 0.923, giving a Δk_{eff} of 0.148 by Eq. (3), or an approximate k_{eff} for the intersection of 0.92 - 0.15 = 0.78, which is to be compared to the calculated value of 0.746. This intersection in contact with the concrete surface ($\Delta k_{eff} \approx 0.13$) would yield the conservative estimate of 0.91 to compare to the calculated k_{eff} of 0.861.

An example of the estimated k_{eff} for a two-arm intersection of $U(100)O_2F_2$ at H:U = 50 with column and arm radii of 6.2 cm and a 3.2-mm-thick steel shell was calculated by the KENO IV Monte Carlo code. The application of Eq. (2) gave 0.80, and the KENO IV result was 0.789 ± 0.006. The same intersection containing $U(93.2)O_2F_2$ at H:U = 50 would be expected to be proportional to their unreflected critical radii as infinite cylinders, because of the correlations of Fig. 6 through 9 and the behavior of data in Fig. 1. The ratio of radii is ~0.97, and the expected k_{eff} would be 0.78 (= 0.8 × 0.97). The KENO IV result for this configuration and fissile material was 0.769 ± 0.006.

Consideration of the subcritical-submerged-intersection data suggests that it is permissible to dispense with the Δk_{eff} margin of 0.05 as compensation for the introduction of container materials. An acceptable upper limit for the k_{eff} of submerged intersections can therefore be taken as 0.9, again consistent with Ref. 7. It may be noted in Table VIII that estimated dimensions corresponding to k_{eff} = 0.9 are about equal to those of similar intersections in Tables IV and V for reflector condition III having k_{eff} = 0.85; this comparison is indicative of similar margins of subcriticality when 3.2-mm-thick steel is introduced as a container material.

The data of Table IX, for $U(5)O_2F_2$ solutions, serve to illustrate the application of an allowance factor[27] to the dimensions of U(93.2) solutions for lower ^{235}U enrichments. The diameter of infinite cylinders of solution with thick water reflectors may be increased by a factor of ~1.6 when the ^{235}U content of the uranium is 5 wt%. If we consider the two-arm intersection in Table VIII with radii of 7.29 cm of $U(93.2)O_2(NO_3)_2$ having a k_{eff} of 1.054, then by the allowance factor, the radii may be increased to 11.66 cm. This is a smaller dimension than either of the two entries in Table IX for two-arm intersections, those with radii of 15.24 and 12.73 cm, confirming a loss in reactivity and the conservatism in the use of the enrichment allowance factor for intersections.

The results presented for plutonium oxide-water mixtures are applicable to plutonium nitrate solutions at the same densities and H:Pu atomic ratios, since the presence of nitrate ions in solutions cause a reactivity loss.

[27] *Nuclear Safety Guide*, TID-7016, Rev. 2, ORNL/NUREG/CSD-6, Oak Ridge National Laboratory (1978).

Two points suggested by this work would benefit ɔm further calculational study:

1. The use of allowance factors applied to infinite cylinders and intersections of uranium of intermediate and low ^{235}U content should be examined for reflector conditions II and III.
2. Cylinders with intersections embedded in concrete should be examined.

The time may have arrived when it would be ɜnsible to allow the concept of nominal reflection ι nuclear criticality safety to fade from use. For the specification of subcritical values, it is ambiguous in application and therefore restrictive.

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