

CRITICALITY
CONTROL OF
FISSILE
MATERIALS

PROCEEDINGS OF A SYMPOSIUM,
STOCKHOLM, 1-5 NOVEMBER 1965



INTERNATIONAL ATOMIC ENERGY AGENCY, VIENNA, 1966

PROCEEDINGS SERIES

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MAXIMUM SAFE LIMITS FOR SLIGHTLY ENRICHED URANIUM AND URANIUM OXIDE*

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Abstract

MAXIMUM SAFE LIMITS FOR SLIGHTLY ENRICHED URANIUM AND URANIUM OXIDE. A survey is given of available critical and exponential data obtained with water-reflected lattices of slightly enriched uranium and uranium oxide rods. Calculations are made for these lattices by an asymptotic multigroup buckling code and a two-group diffusion theory code employing parameters generated by the first code. Comparison between calculations and experiments is made in terms of a k_{eff} , which is the ratio of the calculated k to that calculated from the experimental dimensions and flux traverses and from the calculated migration areas and extrapolation distances. For some of the data, comparisons are also made between the present method of calculation and a more highly sophisticated method. Reasonable care is taken in the present calculations to take account of all important effects; but since the experiments are used to normalize the calculations, absolute accuracy in the calculations is not necessary. A least-squares treatment is given to k_{eff} , the parameter relating calculation and experiment, to obtain an average curve of k_{eff} as a function of the ratio of water to uranium. At low enrichments, k_{eff} is also allowed to vary with rod diameter and enrichment. Nearly all values of k_{eff} lie within ± 0.01 of the average curves. Critical masses and dimensions are calculated with the buckling and two-group codes to correspond to values of k_{eff} lying on the average curves, and safe masses and dimensions are calculated to correspond to values of k_{eff} lying 0.02 away from the average curves. Tables of minimum critical and maximum safe values are presented as a function of enrichment.

INTRODUCTION

By maintaining sufficient limitations on only a few variables, safe operations with fissile materials can be ensured regardless of the values assumed by other variables. Although such limits may be inconveniently small, they are frequently useful because of their simplicity and because the maintenance of less restrictive limits, but on a greater number of variables, may be even more inconvenient. In handling fissile materials, control of the enrichment and form of the fuel is easily achieved. It is also easy to restrict actual or potential moderators and reflectors to those that are no more effective than water. Safe operations can then be ensured by maintaining controls and restrictions on these parameters and by maintaining a mass or a dimensional limit that will always be subcritical regardless of the values of other variables, such as the dimensions of pieces or the actual or potential ratio of moderator to uranium atoms.

Regardless of the degree of sophistication in the method employed to calculate these limits, a sine qua non in nuclear safety work is that the method be normalized to available experimental data. Preferably the

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normalization should be made in terms of some slowly varying parameter relating calculations and critical experiments. Safe mass or dimensional limits are then chosen to correspond to values of the parameter that are subcritical by a sufficiently large margin to compensate adequately for fluctuations in the critical value. For a highly sophisticated method the expectation is that little or no normalization would be required and that it would be difficult in any comparison with experiment to distinguish between errors in calculated results and errors in experiments. Highly sophisticated methods, however, require a large amount of time to set up and a large amount of computing time. Since the number of experimental data points with which to compare is large and since the number of calculations required to establish safe limits is even larger, there is a considerable incentive for using a simple, approximate method of calculation. Provided the experimental data are in the range of interest so that only interpolations or small extrapolations are involved, as is usually the case for slightly enriched uranium, such a procedure is valid and is the one adopted here.

METHOD OF CALCULATION

For a particular lattice, the material buckling is calculated by an asymptotic multigroup transport code and the extrapolation distances by a two group diffusion theory code from parameters generated in the multigroup calculation.

There are twelve energy groups in the buckling code. The top ten groups correspond to Loewenstein and Okrent's [1] top ten groups. Except for the cross-sections of H₂O, the cross-sections for these groups are taken from Yiftah, Okrent and Moldauer [2]. The H₂O cross-sections for these groups and for an eleventh (resonance) group extending from 9.1 keV down to 0.625 eV are derived from a zero dimensional multigroup calculation in which a lethargy width of 0.1 was used and in which allowance was made for anisotropic scattering [3]. The cross-sections in the thermal group are taken from Amster's [4] compendium; intermediate values are obtained by Lagrange interpolation. Homogeneity is assumed in the top ten groups. Hellstrand's [5] resonance integrals for uranium and UO₂ are used for the resonance group, together with effective surface-to-volume ratios calculated on the assumptions of black fuel, uniform source distributions within moderator and cladding, and cosine currents at interfaces; otherwise homogeneity is also assumed in the resonance group. For homogeneous systems, the ²³⁸U resonance integral is expressed as a function of the total scattering cross-section per atom of ²³⁸U. In the thermal group, a P₃ calculation provides the disadvantage factors. Flux and volume weighting of effective hydrogen atoms, ²³⁵U atoms, and 1/v absorbers gives new values of the ratios of ²³⁵U/H and of barns of 1/v absorber per hydrogen atom, the parameters on which the thermal spectrum depends in Amster's compendium. The P₃ calculations are repeated with cross-sections derived from the new spectrum until the process converges.

Leakage in each group, i , is calculated from the asymptotic transport expression

$$D_i B^2 = \frac{B - \Sigma_i \tan^{-1} \frac{B}{\Sigma_i}}{\tan^{-1} \frac{B}{\Sigma_i}} \quad (1)$$

where Σ_i is the transport cross-section. This expression results from replacing the total cross-section by the transport cross-section and assuming isotropic scattering in the laboratory system (transport approximation). This approximation is better [3] than the diffusion approximation (though not as good as indicated in the reference due to b in the reference being too large by a factor of 2). The buckling code calculates by iteration the material buckling required to make the rate of production of neutrons by fission equal the sum of the rates of absorption and leakage. The migration area (used below in the comparison of calculation and experiment) is obtained as $M^2 = (k-1)/B^2$, where k is the ratio of the rates of production and absorption.

The spectrum generated by the calculation furnishes average values of Σ for a fast group comprising the top eleven groups and for a thermal group and average values of c_{11} , c_{21} , c_{12} , and c_{22} where c_{11} represents the number of secondary neutrons per fast interaction remaining in the fast group, c_{21} the number appearing in the slow group, c_{12} the number per slow interaction appearing in the fast group, and c_{22} the number remaining in the slow group. The first three parameters are then adjusted by artificially increasing the number of neutrons released by fission and by increasing the removal cross-section, so that the two group equations, which employ the diffusion approximation for the leakage, $D_i B^2 = B^2 / 3 \Sigma_i$, give the same buckling and the same ratio of fast to slow flux components corresponding to this eigenvalue as are given by the buckling code. The two group parameters for the water reflector are also generated by the buckling code with the buckling set equal to zero. The two group calculations, giving the extrapolation distances into the reflector, are one-dimensional. Separability of the flux is assumed. Dimensions are fixed, and criticality is achieved by adjusting the transverse buckling.

There are a number of places where these procedures obviously could be improved. They account, however, for major effects and are quite fast: a buckling calculation for a simple lattice requires perhaps 20 sec on an IBM-704 and a two region, two group calculation perhaps only 10 sec.

EXPERIMENTAL DATA

A review of data obtained from exponential and critical experiments with lattices of slightly enriched uranium and uranium oxide in water has recently been published [6]. The lattices are of both solid and hollow rods and in some cases boron is present in the moderator. In the present work only the lattices of solid rods in water without boron are considered, and a few of these, for which the experiments appear to be insufficiently well

described, are omitted. The assumption is made that masses or sizes smaller than those achievable with solid rods cannot be achieved with hollow rods or tubes. A study of available data [6] does not appear to contradict this assumption; on the other hand no systematic effort is made here to prove it.

The review presents material bucklings and extrapolation distances (reflector savings), which are inherently not independent. The extrapolation distances were obtained in different ways: sometimes by requiring lattices of different shapes to have common values of geometric buckling and extrapolation distance, sometimes by flux traverses, and sometimes by inference from other experiments or by calculation. In any survey of data made for the purpose of normalizing experiments it is desirable to have bucklings and extrapolation distances all on the same basis. A search was therefore made of the original literature, to which reference is made in the review, to obtain the lattice dimensions. The lattices were assumed to be circular or rectangular cylinders with bases having areas equal to the area of the cell surrounding a rod multiplied by the number of rods. Two group calculations were made with calculated parameters and experimental dimensions to obtain the extrapolation distance on the diameter or on the length and width.

Rod, cladding and cell radii were likewise derived from the original literature, and $V(\text{H}_2\text{O})/V(\text{U})$ ratios were calculated from these radii. There are some small discrepancies between these ratios and the reported ratios. In the compilation in Tables I and II, the laboratory where the data originated is given. No references are given to the original data, however, since the review [6] has a complete listing.

For exponential experiments the axial buckling, B_H^2 , is reported in many of the original references. The Brookhaven National Laboratory results with metal rods, however, generally represent in each case several experiments performed with lattices of various sizes. The original data do not appear to be readily available; hence, a representative lattice diameter of 50.8 cm was assumed, and from the BNL values of extrapolation distance and buckling the axial bucklings of lattices of this diameter were calculated. Geometric bucklings are then the sum of these axial bucklings and radial bucklings calculated by the two group code for 50.8 cm diameter cylinders. For the BNL data obtained with oxide rods a lattice diameter of 36.00 cm was assumed. The original reference gives results obtained with both variable loading and radial flux traverse, and both are included here.

For critical experiments, the reported axial extrapolation distances (and hence axial bucklings) are often used here because conditions at the ends of the rods make calculations uncertain. In some cases water did not cover the ends and in others, where it did, end fittings on the rods changed the nature of the reflector. Where water covered the ends and no end fittings were involved, two-group calculations were made to obtain the axial extrapolation distance.

In many of the Hanford Laboratory experiments the fuel rods were encased in plastic tubes. The experimenters reported that results differed insignificantly from those obtained with unclad fuel rods in water. In calcu-

TABLE I
Lattices of Metal Rods in Water

Matls ^(a)	Cell Rad11 ^(b) , cm			V_{H_2O}/V_U	Lattice Type and Dim. ^(c) , cm	B_H^2 , m ⁻²		Δk_{eff} x 100	Laboratory
						(H, cm)	or (H, cm)		
AW	0.660	1.032		1.44	S	120.00	120.00	-14.70	Harwell
		1.146		2.01	120.00	120.00	-13.36	-0.41	
		1.433		3.71	120.00	120.00	-20.68	0.34	
AW	1.000	1.526		1.33	T	113.28	117.70	-17.36	ORNL
		1.686		1.84	125.05	130.19	-11.89	-0.55	
		1.778		2.16	132.03	137.10	-12.40	-0.38	
AW	1.175	2.134		1.53	T	34.42		-94.26	Hanford
		2.000		1.90	36.88		-88.50	1.04	
		2.134		2.30	39.34		-85.26	1.17	
AVQW	1.397	1.524	1.651	2.373	1.49	T	50.80		BNL
AW	1.499	2.501		1.79	T	157.58	157.58	-10.62	ORNL
		2.301		1.36	168.80	168.80	-8.81	-0.45	
AW	1.524	2.150		0.99	S	120.00	120.00	-14.70	Harwell
		2.645		2.01	120.00	120.00	-13.36	1.48	
AWQW	1.626	1.645	1.746	2.293	S	82.98	86.65	-35.61	AECL
		3.024		2.33	87.10	89.81	-38.64	-1.09	
BVRW	0.953	0.969	1.022	0.85	S	63.68	72.26	-28.91	Harwell
		1.648		1.84	66.08	71.74	-2.16	-0.41	
GVQW	1.524	1.544	1.594	0.59	S	63.09	63.09	-42.28	Harwell
		2.150		0.90	88.62	72.18	-11.57	-0.52	
		2.408		1.40	72.78	71.38	-3.26	-0.75	
CVSW	1.524	1.546	1.594	1.93	S	91.12	71.88	-1.13	Harwell
		2.150		0.90	53.34	53.34	-44.44	-0.96	
		2.651		1.93	61.09	61.09	-27.70	-0.92	
DVQW	1.697	1.786	1.910	2.667	T	39.56		-57.31	Hanford
		2.801		1.46	41.54		-50.02	0.12	
		2.934		1.72	43.52		-47.50	-0.15	
DVTW	1.697	1.793	1.918	3.201	T	47.48		-46.59	Hanford
		2.667		1.19	39.56		-79.44	0.07	
		2.801		1.45	41.54		-70.74	0.91	
EQW	1.175	1.264		2.934	T	43.52		-69.10	Hanford
		1.680		0.89	47.48		-69.33	0.79	
		1.867		1.37	30.98		-89.34	1.02	
EVEV-QW	1.175	1.194	2.126	0.86	T	35.22		-62.20	Hanford
		2.197		1.37	34.42		-65.35	0.02	
		3.267		1.74	37.74		-50.02	0.31	
FVQW	0.318	0.330	0.402	0.560	T	50.80		-46.59	BNL
		0.602		1.99	50.80		-48.36	-0.80	
		0.681		2.99	50.80		-47.90	0.10	
FVQW	0.492	0.504	0.575	0.751	T	54.74		-29.28	Hanford
		0.757		1.00	50.80		-29.28	0.30	
		0.833		1.50	50.80		-12.45	0.14	
FVQW	0.762	0.775	0.846	0.902	T	67.18		-12.76	BNL
		1.028		2.00	60.80		-39.51	-0.03	
		1.139		3.00	50.80		-31.83	0.20	
FQW	0.953	1.029		1.138	T	50.80		-28.41	BNL
		1.260		1.50	50.80		-32.18	-0.01	
		1.370		2.00	50.80		-46.77	0.00	
GVQW	1.524	1.541	1.590	0.833	T	50.80		-4.67	Harwell
		2.130		1.50	50.80		-32.14	-0.43	
		2.408		1.41	50.80		-32.14	-0.43	
HVQW	0.318	0.330	0.402	0.560	T	50.80		-25.00	BNL
		0.602		1.99	50.80		-24.11	-0.37	
		0.681		2.99	50.80		-31.54	-0.69	
FQW	0.953	1.029		1.138	T	50.80		-24.11	BNL
		1.260		1.50	50.80		-31.54	-0.69	
		1.370		2.00	50.80		-39.66	-0.24	
GVQW	1.524	1.541	1.590	0.90	S	53.34	53.34	-23.90	Harwell
		2.408		1.41	39.30	70.90	-5.44	-0.71	
		2.651		1.94	42.29	42.29	-20.29	-0.21	
HVQW	0.318	0.330	0.402	0.560	T	50.80		-30.42	BNL
		0.602		1.99	50.80		-20.99	0.66	
		0.681		2.99	50.80		-15.51	0.67	
				0.751	3.99	50.80		-18.20	0.51

TABLE I, Continued

Matls ^(a)	Cell Radii ^(b) , cm			V_{H_2O}/V_U	Lattice Type and Dim. ^(c) , cm	$B_{H_2}^2$, m ⁻² or (H, cm)	Δk_{eff} x 100	Laboratory	
HVQW	0.492	0.504	0.575	1.00	T 50.80	-38.00	0.36	BNL	
			0.833	1.50	50.80	-21.22	0.10		
			0.902	2.00	50.80	-12.42	0.31		
			1.028	3.00	50.80	-9.56	0.37		
HVQW	0.762	0.775	0.846	1.139	4.00	50.80	-15.64	0.21	BNL
			1.138	1.00	50.80	-30.20	0.08		
			1.260	1.50	50.80	-13.20	-0.06		
			1.370	2.00	50.80	-6.53	0.09		
			1.567	3.00	50.80	-9.07	0.13		
IVQW	0.762	0.775	0.846	1.743	4.00	50.80	-21.05	-0.17	BAPL
				1.374	2.02	59.12	(121.92)	-0.01	
JVQW	0.762	0.775	0.846	1.570	3.01	61.20	(121.92)	0.14	BAPL
				1.263	1.52	55.22	(121.92)	0.09	
KVQW	0.492	0.504	0.575	1.374	2.02	50.28	(121.92)	0.23	BAPL
				1.570	3.01	51.20	(121.92)	0.18	
				0.906	2.03	54.48	5.34	0.33	
LVQW	0.492	0.504	0.575	0.906	2.03	56.34	7.85	0.33	BNL
				1.030	3.02	51.74	5.32	0.31	
				0.757	1.00	50.80	-29.24	0.30	
				0.833	1.50	50.80	-11.11	-0.04	
LVQW	0.762	0.775	0.846	0.902	2.00	50.80	-1.04	0.17	BNL
				1.028	3.00	50.80	3.96	0.30	
				1.139	4.00	50.80	-1.05	0.12	
				1.138	1.00	50.80	-19.92	0.25	
				1.260	1.50	50.80	-1.75	-0.03	
MVQW	1.697	1.781	1.905	1.370	2.00	50.80	6.13	0.11	Hanford
				1.567	3.00	50.80	4.87	0.07	
				1.743	4.00	50.80	-5.95	-0.11	
				2.667	1.21	37.34	-30.09	0.24	
				2.801	1.46	40.40	-22.23	-1.03	
NW	0.762	1.334	1.905	2.934	1.73	41.08	-17.15	-0.11	Hanford
				3.201	2.30	44.80	-15.99	-0.26	
				3.467	2.92	48.54	-22.04	-0.19	
				2.06	2.06	27.84	-27.38	-0.03	
				2.06	2.06	35.42	(81.28)	0.07	
				2.06	2.06	42.56	(40.64)	0.13	
				2.71	2.71	27.06	-29.60	0.09	
				2.71	2.71	34.58	(81.28)	0.10	
				2.71	2.71	41.28	(40.64)	0.15	
				3.41	3.41	28.44	-26.82	-0.31	
				3.41	3.41	35.68	(81.28)	-0.03	
				3.41	3.41	42.76	(40.64)	0.10	
				4.18	4.18	30.80	-21.92	-0.31	
				4.18	4.18	38.08	(81.28)	-0.06	
				4.18	4.18	46.40	(40.64)	0.08	
				5.18	5.18	34.92	-16.30	-0.14	
				5.18	5.18	42.84	(81.28)	0.01	
5.18	5.18	54.14	(40.64)	0.16					
6.84	6.84	44.56	-11.93	-0.13					
6.84	6.84	56.34	(81.28)	0.15					
NW	1.175	2.000	2.000	1.90	1.90	29.72	-20.62	-0.22	Hanford
				1.90	1.90	36.32	(81.28)	-0.01	
				1.90	1.90	43.40	(40.64)	0.31	
				2.30	2.30	29.94	-21.63	-0.63	
				2.30	2.30	36.34	(81.28)	-0.03	
				2.30	2.30	43.46	(40.64)	0.25	
				2.72	2.72	29.74	-26.06	-0.35	
				2.72	2.72	37.52	(81.28)	-0.07	
				2.72	2.72	45.32	(40.64)	0.17	
				3.18	3.18	31.48	-23.70	-0.26	
				3.18	3.18	39.66	(81.28)	-0.02	
				3.18	3.18	48.52	(40.64)	0.26	
				3.90	3.90	36.40	-17.35	-0.37	
				3.90	3.90	44.86	-7.13	0.14	
3.90	3.90	56.74	(81.28)	0.17					
3.90	3.90	56.74	(40.64)	0.64					

TABLE I, Continued

Matls(a)	Cell Radii(b), cm		V_{H_2O}/V_U	Lattice Type and Dim.(c), cm	B_{eff}^2 , m ⁻²	Δk_{eff}	Laboratory					
					or (H, cm)	x 100						
OW	0.222	0.400	2.24	T	22.36	-50.95	-0.46	Hanford				
		0.500	4.06		25.72	-6.78	-0.16					
		0.500	4.06		29.56	(59.69)	-0.02					
		0.600	6.29		24.98	-16.98	-0.01					
		0.600	6.29		30.08	(59.69)	0.24					
		0.667	8.00		25.96	-18.90	-0.01					
		0.667	8.00		31.84	(59.69)	0.24					
		0.734	9.89		28.56	-13.55	-0.08					
		0.734	9.89		34.54	(59.69)	0.17					
		0.800	11.96		31.16	-12.59	-0.13					
		0.800	11.96		38.30	(59.69)	0.04					
		OW	0.381		0.667	2.06	T		22.68	-39.66	0.33	Hanford
					0.800	3.41			25.46	-5.20	-0.04	
					0.800	3.41			27.64	(81.28)	0.12	
0.800	3.41			31.50	(40.64)	0.37						
0.934	5.00			25.54	-9.54	-0.27						
0.934	5.00			28.32	(81.28)	-0.09						
0.934	5.00			32.18	(40.64)	0.32						
1.067	6.84			27.24	-10.03	-0.17						
1.067	6.84			30.18	(81.28)	0.36						
1.067	6.84			35.18	(40.64)	0.25						
1.200	8.93			30.64	-8.13	-0.31						
1.200	8.93			34.28	(81.28)	-0.02						
1.200	8.93			40.58	(40.64)	0.16						
OW	0.762			1.334	2.06	T		26.26	-6.49	-0.20	Hanford	
		1.334	2.06	28.60	(81.28)		0.24					
		1.334	2.06	32.90	(40.64)		0.41					
		1.467	2.71	25.06	-10.82		-0.13					
		1.467	2.71	28.02	(81.28)		0.01					
		1.467	2.71	31.94	(40.64)		0.31					
		1.600	3.41	26.20	-7.86		-0.67					
		1.600	3.41	28.60	(81.28)		-0.16					
		1.600	3.41	32.72	(40.64)		0.08					
		1.734	4.18	27.08	-8.13		-0.36					
		1.734	4.18	29.84	(81.28)		-0.06					
		1.734	4.18	34.48	(40.64)		0.05					
		1.894	5.18	29.58	-5.92		-0.43					
		1.894	5.18	32.54	(81.28)		-0.20					
		1.894	5.18	37.94	(40.64)		0.08					
		2.134	6.84	34.40	-6.97		-0.17					
		2.134	6.84	38.74	(81.28)		0.13					
		2.134	6.84	47.14	(40.64)		0.40					
		OVW	1.175	1.194	1.87		T	31.00	(60.96)	0.00		Hanford
				2.000	1.87			33.96	(40.64)	0.48		
2.134	2.27			30.90	(60.96)	-0.18						
2.134	2.27			34.02	(40.64)	0.13						
2.267	2.69			31.80	(60.96)	-0.50						
2.267	2.69			34.70	(40.64)	0.17						
2.401	3.14			32.88	(60.96)	-0.20						
2.401	3.14			36.16	(40.64)	0.34						
2.601	3.87			28.96	-17.85	-0.14						
2.601	3.87			28.96	-16.12	0.23						
2.601	3.87			36.26	(60.96)	-0.17						
2.601	3.87			40.42	(40.64)	0.35						

(a) Materials used in the calculations are designated as follows:

A - O are uranium, Q - U are cladding, V is void, and W is water. The various weight % ²³⁵U and metal densities are:
 A - 0.71, 18.9; B - 0.923, 18.8; C - 0.923, 18.7;
 D - 0.95, 18.9; E - 1.007, 18.9; F - 1.027, 18.9;
 G - 1.136, 18.7; H - 1.143, 18.9; I - 1.145, 18.9;
 J - 1.294, 18.9; K - 1.296, 18.9; L - 1.299, 18.9;
 M - 1.44, 18.9; N - 2.00, 18.9; O - 3.063, 18.9.
 The various cladding materials, percentage compositions,
 and densities are Q - Al, 100 Al, 2.7; R - Al, 100 Al, 2.8;
 S - SS, 74 Fe, 18 Cr, 8 Ni, 8.2; T - Fe, 100 Fe, 7.86.

(b) Radii are outer radii of regions, beginning with uranium and ending with equivalent cylindrical cell radius.

(c) Lattice type is either square (S) or triangular (T). Dimensions are length and width of a rectangular array if two are given or diameter of a circular array if only one is given.

TABLE II
Lattices of Oxide Rods in Water

Matls.(a)	Cell Radit.(b), cm			V_{H_2O}/V_U	Lattice Type and Dim.(c), cm	B_H^2 , m ⁻² or (H, cm)	Δk_{eff} x 100	Laboratory		
AVPW	0.493	0.504	0.575	0.818	3.97	T 90.28	5.11	0.50	BAPL	
				0.818	3.97	96.52	7.21	0.46		
				0.868	4.96	91.60	5.24	0.69		
AVPW	0.763	0.775	0.846	0.868	4.96	101.44	8.30	0.62	BAPL	
				1.158	3.06	82.50	5.05	-0.06		
				1.158	3.06	85.70	6.51	-0.06		
				1.158	3.06	92.38	8.76	-0.22		
				1.239	4.01	79.42	5.13	-0.01		
				1.239	4.01	83.78	7.32	-0.01		
				1.239	4.01	96.88	11.93	-0.17		
BVPW	0.486	0.504	0.575	1.319	5.01	82.88	5.20	0.13	BAPL	
				1.319	5.01	88.88	7.88	0.13		
				0.818	2.91	76.26	5.24	-0.11		
				0.818	2.91	79.56	7.06	-0.14		
				0.818	2.91	81.66	8.13	-0.15		
				0.818	2.91	85.22	9.61	-0.25		
				0.868	3.64	72.72	5.29	-0.01		
				0.868	3.64	75.26	7.02	0.00		
				0.868	3.64	90.42	14.22	-0.17		
				0.948	4.89	75.24	5.30	0.24		
CVPW	0.565	0.572	0.635	0.948	4.89	78.42	7.20	0.21	GEND	
				0.948	4.89	93.48	13.95	0.11		
				0.896	2.59	33.34	87.32	7.05		-0.75
				0.896	2.59	34.92	87.32	9.54		-0.78
DVPW EVQW	0.515	0.522	0.603	1.267	7.81	36.52	87.32	5.93	B and W	
				1.267	7.81	34.92	87.32	8.34		1.02
				0.853	2.86	83.28	4.07	-0.43		
	EVQW	0.381	0.389	0.430	0.599	2.21	64.02	5.42	-0.04	WAPD
					0.599	2.53	65.58	12.35	-0.21	
					0.599	2.53	65.52	13.41	-0.30	
					0.623	2.92	53.64	5.42	-0.08	
					0.674	3.90	48.54	5.42	0.02	
					0.707	4.56	50.30	10.96	-0.06	
					0.707	4.56	52.60	6.64	-0.02	
					0.821	7.11	47.20	5.53	0.34	
					0.881	8.59	49.54	5.47	0.39	
FRW	0.564	0.635	0.877	0.953	10.50	54.34	5.41	0.30	B and W	
				2.61	2.61	53.40	(168.90)	0.14		BNL
				2.93	2.93	36.00	-30.97	0.24		
				0.950	3.56	47.60	(168.90)	-0.08		B and W
				0.955	3.64	36.00	-26.35	0.17		
				1.028	4.66	36.00	-28.05	-0.34		BNL
				1.139	6.38	36.00	-22.96	0.07		
				1.139	6.38	36.00	-23.91	-0.20		
				1.139	6.38	36.00	-23.96	0.12		
				1.240	8.10	48.40	(168.90)	-24.67		B and W
1.295	9.08	36.00	-34.73	-0.18						
GVSW	0.377	0.389	0.430	0.599	2.53	52.72	-35.97	-0.53	WAPD	
				0.797	4.57	38.02	18.57	-0.24		
HRW	0.564	0.604	0.818	0.853	2.18	40.36	8.85	-0.14	B and W	
				0.853	2.59	37.50	3.60	-0.30		
HRW	0.564	0.635	0.877	0.950	3.57	38.60	(168.90)	-0.62	B and W	
				1.240	8.10	37.20	(168.90)	-0.44		
				1.343	10.00	39.60	(168.90)	-0.36		
				1.343	10.00	39.60	(168.90)	-0.53		

(a) Materials used in the calculations are designated as follows:

A - H are uranium oxide, P - U are cladding, V is void and W is water. The various weight % ²³⁵U, weight % oxygen, and oxide densities are: A - 1.311, 11.85, 7.52 - 7.53; B - 1.311, 11.85, 10.53; C - 1.85, 12.00, 10.35; D - 2.459, 11.87, 10.29; E - 2.700, 11.90, 10.18; F - 3.000, 11.85, 9.45; G - 3.700, 11.90, 10.37; H - 4.020, 12.00, 9.46. The various cladding materials, percentage compositions, and densities are P - Al, 100 Al, 2.7; Q - SS, 72.22 Fe, 18.20 Cr, 9.58 Ni, 0.003B, 7.9; R - SS, 70.90 Fe, 18.50 Cr, 10.60 Ni, 0.002B, 7.9; S - SS, 72.02 Fe, 18.54 Cr, 9.44 Ni, 0.005B, 7.9. Boron was not actually present in the stainless steel but was used to mock up absorbers other than Fe, Cr, and Ni.

(b) See note (b) of Table I.

(c) V_{H_2O}/V_U is the ratio of the volume of water to the volume of uranium having a density of 18.9 g/cm³.

(d) See note (c) of Table I.

Calculations for these lattices the plastic tubes were therefore assumed to be water.

Comparison between calculation and experiment is made in terms of

$$k_{\text{eff}} = \frac{1 + M^2 B_{\text{calc.}}^2}{1 + M^2 B_{\text{expt.}}^2} \quad (2)$$

where $B_{\text{expt.}}^2$ is obtained from experimental dimensions, calculated extrapolation distances, and (for exponentials and some criticals) experimental axial bucklings. The metal rod data for enrichments up through 1.5% are considered as a group. Other groups consist of metal rod data at 2 wt. % ^{235}U , at 3.063 wt. %, and all the oxide rod data. The k_{eff} values relating calculation and experiment in all cases show a trend toward low values at low ratios of water to uranium (e.g. Fig. 1). The metal rod data below 1.5 wt. % also show trends with enrichment and rod diameter.

For the metal rod data up through 1.5 wt. %, a least squares fit was made to a polynomial in $V(\text{H}_2\text{O})/V(\text{U})$ and to linear terms in rod diameter and enrichment. Since all the data are undoubtedly not equally good and since some (e.g. the BNL data) are the result of several determinations, a least-squares fitting without the use of appropriate weighting factors is not a strictly valid procedure. It does, however, serve the useful purpose of providing an average curve through the data expressed in terms of k_{eff} .

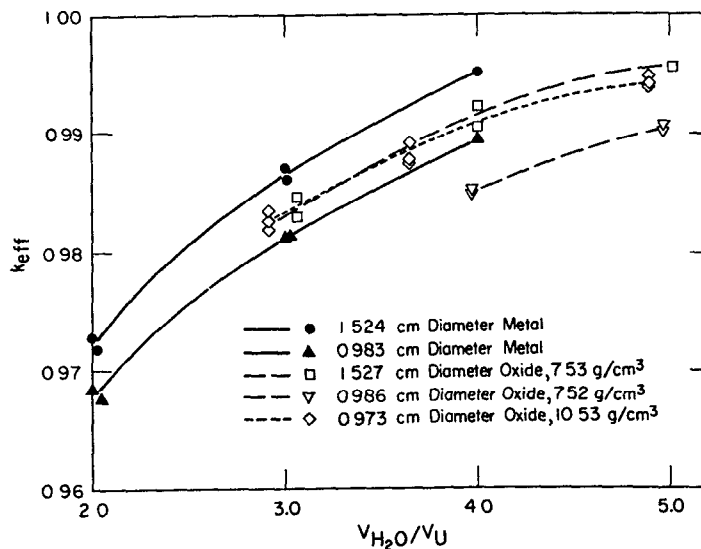


FIG. 1. Metal and oxide rod data at an enrichment of about 1.3 wt. %.

The curve obtained was plotted against $V(\text{H}_2\text{O})/V(\text{U})$, a smooth extrapolation was made to values of $V(\text{H}_2\text{O})/V(\text{U})$ somewhat higher than the range encompassed by the data, and a polynomial fit was made to the resulting extrapolated curve. Deviations from this curve and from the linear dependence of k_{eff} on rod diameter and enrichment are given in Table I and

fall within ± 0.015 and for the most part within ± 0.01 . The deviations were obtained by subtracting the value relating calculation and experiment from the average value at the same $V(\text{H}_2\text{O})/V(\text{U})$, rod diameter, and enrichment; hence, a negative deviation indicates that the average value overestimates the reactivity and is slightly conservative.

The metal rod data at 2 wt. % and at 3.063 wt. % ^{235}U were fitted by least squares to polynomials in $V(\text{H}_2\text{O})/V(\text{U})$ and linear terms in rod diameter. The variation with rod diameter is in the opposite direction from that for the lower enrichments. Again curves were plotted against $V(\text{H}_2\text{O})/V(\text{U})$, smooth extrapolations were made to higher ratios, and polynomial fits were made to the extrapolated curves.

The oxide rod data appear to show no significant trends with rod diameter or enrichment, possibly because the data are not so extensive as for metal rods. Again least-squares fitting gave an average curve as a function of $V(\text{H}_2\text{O})/V(\text{U})$. To extend the curve to high ratios, use was made of critical data obtained with aqueous solutions of UO_2F_2 in which the uranium contained 4.89 wt. % ^{235}U [7]. In calculations made for these solutions, fluorine was assumed to have the same nuclear properties as oxygen. Deviations from the average curve are presented in Table II for the oxide lattices and in Table III for the solution data [7] that have been published in the open literature.

TABLE III

Water-Reflected Cylinders of Aqueous Solutions of UO_2F_2
in which the Uranium Contains 4.89% ^{235}U [7]

Conc g U/l	$V_{\text{H}_2\text{O}}/V_{\text{U}}$	Dis., cm	Ht, cm	$\Delta k_{\text{eff}} \times 100$
890	18.18	30.7	173.2	0.79
		31.1	139.6	0.64
		33.0	72.4	0.64
		38.1	41.7	0.52
870	18.68	38.1	44.8	0.02
		38.1	50.4	0.00
		50.8	29.3	-0.10
		38.1	75.6	-0.22
728	22.98	50.8	34.7	-0.33
		38.1	153.0	-0.20
650	26.14	50.8	40.1	-0.22
		50.8	85.7	-0.08
496	35.25	68.8 (sphere)		-0.06
452	38.99			

Experiments [8, 9] with homogeneous systems have been performed to determine the amount of neutron absorber that must be added or subtracted to make $k_{\infty} = 1$ and $B_M^2 = 0$. Calculations were made of the bucklings of these systems, with the absorber considered to be boron. The k_{eff} relating calculation and experiment is then simply the calculated k . For systems [8] having enrichments near 1 wt. %, deviations of the calculated k_{eff} values from the average values (extrapolated to zero rod diameter) obtained by fitting the metal rod data in the range from 0.71 to 1.5 wt. % are given in Table IV. Deviations of k_{eff} 's calculated for the homogeneous system [9] having an enrichment of 3.04 wt. % from the average curve for oxide lattices are all negative (conservative), and in absolute value less than 0.02 for $V(\text{H}_2\text{O})/V(\text{U})$ ratios less than 9. For ratios from 20 to 35, however, the

TABLE IV
Infinite Critical Compositions/87

Wt % ^{235}U	$V_{\text{H}_2\text{O}}/V_{\text{U}}$	kg B/g U	$\Delta k_{\text{eff}} \times 100$
1.0059	2.741	-5.26	-0.20
	3.622	-5.12	-0.08
	4.456	-8.82	0.26
	4.970	-12.88	0.50
1.0704	5.378	-21.79	0.05
	2.705	4.30	-0.80
	4.175	4.57	-0.64
	5.106	-3.23	-0.34
1.1586	2.712	21.45	-1.08
	4.287	25.20	-0.69
	4.938	24.48	-0.14
	5.374	16.60	-0.48

TABLE V
Analyses of BNL Data Obtained
with 0.635, 0.983, and 1.524 cm Dia. Rods

% ^{235}U	Rod Dia., cm	$V_{\text{H}_2\text{O}}/V_{\text{U}}$	$\Delta k = k_c \left[\frac{1 + M^2 B_{\text{calc}}^2}{1 + M^2 B_{\text{exptl}}^2} \right]^{(a)}$		
			$\Delta k \times 10^2$	$\Delta k' \times 10^2$	$\Delta k'' \times 10^2$
1.027			0.365	0.674	-0.260
1.143			0.851	1.160	0.278
1.299			0.775	1.125	0.114
	1.524		0.557	0.875	-0.117
	0.983		0.584	0.872	0.025
	0.635		0.588	1.341	0.301
		1.0	0.430	0.985	0.125
		1.5	0.756	1.213	-0.064
		2.0	0.724	1.059	0.116
		3.0	0.588	0.771	0.101
		4.0	0.708	0.835	-0.119

(a) For Δk and $\Delta k'$, $k_c = 1$. For $\Delta k''$, k_c is average value as a function of $V_{\text{H}_2\text{O}}/V_{\text{U}}$, % ^{235}U , and rod diameter. For Δk and $\Delta k'$, B_{calc}^2 is taken from Reference 10. For $\Delta k''$, B_{calc}^2 is calculated by the procedure outlined here. For Δk , B_{exptl}^2 is the reported value. For $\Delta k'$ and $\Delta k''$, B_{exptl}^2 is the reported value adjusted to extrapolation distance calculated here for a 50.8 cm diameter cylinder.

magnitude of the deviation increases from about 0.03 to 0.06 and appears inconsistent with the results obtained with the UO_2F_2 solutions.

Close examination of Tables I and II no doubt will show small trends that separable variations with $V(\text{H}_2\text{O})/V(\text{U})$, rod diameter, and enrichment make no allowance for. A more sophisticated analysis of the BNL exponential data with metal rod lattices has been reported [10]. Bucklings calculated in this analysis are compared in Table V both with reported experimental bucklings [6] and with those adjusted to the extrapolation distances calculated here for 50.8 cm diameter cylinders. The comparison is made in terms of average deviations of k_{eff} as calculated by Eq. (2) from unity. The migration areas used are those calculated here, but the k_{eff} values in the first column differ insignificantly from reported values [10]. The averages are taken over the two unspecified variables. The third column lists similar averages of the deviation given in Table I. All three columns indicate the rods enriched to 1.027 wt. % to be less reactive than

would be expected from normalization to the experimental results at 1.143 and 1.299 wt. % enrichment. Despite the linear variation with rod diameter allowed in the third column, all three columns indicate the 0.635 cm diameter rods to be more reactive than would be expected from normalization to the experimental results for the larger diameter rods.

CRITICAL AND SAFE VALUES

Critical diameters of spheres and of infinite cylinders and critical thicknesses of infinite slabs of lattices of unclad metal and oxide rods in water were calculated by the buckling and two-group codes to be those having the average values of k_{eff} relating experiment and calculation. Safe dimensions were calculated to be those having values of k_{eff} that are 0.02 smaller. Wide ranges of $V(H_2O)/V(U)$ and of rod diameter were employed to determine the minima that are given in Tables VI, VII, and VIII. The tables also give the approximate rod diameter and $V(H_2O)/V(U)$ ratio at which the minima occur so that reference to Tables I-IV can be made to determine the amount of extrapolation outside the range of the experimental data.

TABLE VI
Minimum Critical and Maximum Safe Masses for
Uranium Metal (density 18.9 g/cm³) and
Uranium Oxide (density 10.9 g/cm³, 12% oxygen) in Water

Wt % ²³⁵ U	Metal or Oxide	Mass, kg ²³⁵ U		Approximate	
		Critical	Safe	Rod Dia., cm	V_{H_2O}/V_U
0.71	M	49,200.	260.	2.0	1.8
0.80	O	725.	143.	2.8	2.2
0.95	O	42.0	26.2	2.0	3.0
1.00	M	18.2	13.1	1.3	2.8
1.08	O	19.4	13.9	1.7	3.5
1.17	O	13.8	10.4	1.5	3.7
1.30	O	9.70	7.65	1.3	4.0
1.40	M	6.23	5.04	0.8	4.5
1.50	O	6.61	5.36	1.0	5.0
2.00	M	3.48	2.94	0.5	6.0
2.00	O	3.99	3.37	0.9	6.0
3.00	M	2.36	2.03	0.3	10.0
3.00	O	2.45	2.12	0.5	10.0
5.00	M	1.47	1.29	0.13	18.0
5.00	O	1.56	1.36	0.3	18.0

The average k_{eff} 's used for both metal and oxide rods at enrichments not in excess of 1.5 wt. % are those obtained by fitting the metal rod data. Available, comparable metal and oxide rod data expressed as k_{eff} have approximately the same variation with rod diameter and $V(H_2O)/V(U)$ (see Fig. 1), and lattices of normal density oxide rods and of metal rods of the same diameter have approximately the same k_{eff} at the same $V(H_2O)/V(U)$ ratio. The good agreement (Table IV) of the homogeneous system data with the average k_{eff} values is a further indication of the validity of this approach.

For metal rods at 2 and 3 wt. %, the average k_{eff} values are those obtained by fitting the Hanford data at 2 and 3.063 wt. %. For oxide rods at 2, 3, and 5 wt. % and for metal rods at 5 wt. % the average values are those

TABLE VII

Minimum Critical and Maximum Safe Sizes for
Uranium Metal (density 18.9 g/cm³) and
Uranium Oxide (density 10.9 g/cm³, 12% oxygen) in Water

Wt % ²³⁵ U	Metal or Oxide	Cylinder Dia., cm		Slab Thick- ness, cm		Volume, liter		Approximate	
		Critical	Safe	Critical	Safe	Critical	Safe	Rod Dia., cm	V _{H₂O} /V _U
0.71	M	94.9	163.	616.	102.	1,010,000.	5320.	2.0	1.8
0.80	O	250.	147.	159.	91.2	19,000.	3920.	3.0	2.0
0.95	O	94.1	80.6	56.8	48.0	1,070.	686.	2.5	2.5
1.00	M	61.9	55.5	35.9	31.6	321.	234.	1.8	2.0
1.08	O	71.0	63.6	41.7	36.9	476.	348.	2.3	2.5
1.17	O	62.4	56.8	36.1	32.4	328.	251.	2.0	3.0
1.30	O	54.3	50.1	30.8	28.0	221.	176.	1.8	3.0
1.40	M	41.1	38.2	22.2	20.2	101.	82.5	1.4	2.4
1.50	O	46.5	43.4	25.7	23.6	143.	118.	1.6	3.5
2.00	M	32.0	30.1	16.2	15.0	50.5	43.0	1.1	3.0
2.00	O	36.1	34.0	18.8	17.4	70.8	60.3	1.5	3.5
3.00	M	26.3	24.9	12.5	11.5	29.7	25.8	0.9	3.5
3.00	O	28.8	27.4	14.1	13.0	38.2	33.4	1.2	4.0
5.00	M	21.0	20.0	8.98	8.22	16.5	14.5	0.7	4.0
5.00	O	23.4	22.3	10.6	9.79	22.1	19.5	0.9	6.0

TABLE VIII

Minimum Critical and Maximum Safe Areal Densities
for Slightly Enriched Uranium Metal (density 18.9 g/cm³)
and Uranium Oxide (density 10.9 g/cm³, 12% oxygen) in Water

Wt % ²³⁵ U	Metal or Oxide	Areal Density, g ²³⁵ U/cm ²		Approximate	
		Critical	Safe	Rod Dia., cm	V _{H₂O} /V _U
0.71	M	30.0	4.85	1.9	2.0
0.80	O	5.90	3.17	2.5	2.5
0.95	O	2.07	1.69	1.5	3.5
1.00	M	1.72	1.47	0.8	4.0
1.08	O	1.56	1.33	1.3	4.5
1.17	O	1.35	1.17	1.0	5.5
1.30	O	1.13	1.00	0.9	6.0
1.40	M	1.06	0.97	0.5	6.5
1.50	O	0.95	0.86	0.9	6.0
2.00	M	0.82	0.75	0.3	11.0
2.00	O	0.847	0.772	0.5	11.0
3.00	M	0.70	0.64	0.2	17.0
3.00	O	0.666	0.612	0.3	19.0
5.00	M	0.55	0.51	0.08	30.0
5.00	O	0.553	0.512	0.1	32.0

obtained by fitting all the oxide data and the UO₂F₂ solution data. No metal rod data at 5 wt. % are available. The metal rod data at 2 and 3.063 wt. %

deviate respectively by $-(0.0082 \begin{smallmatrix} +0.0099 \\ -0.0122 \end{smallmatrix})$ and $-(0.0103 \begin{smallmatrix} +0.0147 \\ -0.0100 \end{smallmatrix})$ from

the average k_{eff} values obtained by fitting the oxide rod and solution data, i. e. at 2% and 3.063 wt. % the metal rod lattices tend to be less reactive than would be expected from normalization to the oxide rod lattices.

A similar listing of critical parameters for slightly enriched uranium metal rods in water has been made by Brown [11]. His values at enrichments of 2 and 3.063 wt. % are in good agreement with those calculated here, but at 1.03 wt. % and even more so at 5 wt. % his values are larger.

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DISCUSSION

K.H. PUECHL: I have a question regarding your calculation procedure. Since Amster's compilation is based on homogeneous mixtures for various ²³⁵U/H ratios, etc., and since one requires averaged cross-sections for the P₃ disadvantage factor calculation, one must obviously iterate between the P₃ calculation and the Amster compilation to come up simultaneously with effective number densities and appropriate average cross-sections. Was such iteration performed?

H.K. CLARK: Yes, it generally took about three iterations for satisfactory conversion.

C.S. PASUPATHY: I have two questions. First, did you in your calculations try any models other than the Wigner-Wilkins, for example Nelkin or Koppel-Young?

H.K. CLARK: No, we did not. Of course the Wigner-Wilkins model is far from exact, but as long as one can tie one's results in with experiment this kind of empirical approach seems acceptable. One gets a slowly varying parameter, on the basis of which one can check inaccuracies and make the desired interpolations.

C.S. PASUPATHY: Secondly, are the UO₂ densities given in your table actual values or have they been increased to those of the metal in the manner you indicated in your oral presentation?

H.K. CLARK: Actual UO₂ densities are used. Volume ratios of water to uranium for oxide lattices, however, are calculated as though the uranium had a density of 18.9 g/cm³. The tables of critical and safe values for oxide lattices assume an oxide density of 10.9 g/cm³.

E.D. CLAYTON (Chairman): Would you care to comment on the relative merits of specifying a k_{eff} as an index of nuclear safety and using a fraction of the critical dimension or fraction of the critical volume?

H.K. CLARK: It is important that the margin of safety be expressed in terms of the parameter used to relate calculations and experiments. Particularly in the range of low enrichments, where many of the experiments are exponential, k_{eff} is a natural parameter to use and fraction of critical mass would not have much meaning because critical masses, or dimensions, may not have been determined.

E.D. CLAYTON: About what difference does an uncertainty of ± 0.02 for k_{eff} make in the minimum water-reflected spherical critical masses of uranium rods in the 1 to 2 wt. % ^{235}U enrichment range?

H.K. CLARK: I don't recall the uncertainty in mass that corresponds to an uncertainty of 0.02 in k_{eff} . It is, however, much larger at an enrichment of 1 than 2 wt. %.