

Effect of Distribution of Fissile Material on Critical Mass*

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The influence of the distribution of ^{235}U , ^{239}Pu , and ^{233}U in water on the minimum critical mass, the minimum critical concentration per unit length of cylinder, and the minimum critical mass per unit area of slab surface is calculated. Two energy groups are employed in the calculations with parameters normalized to force agreement between calculations and experiments performed with water-reflected spheres containing uniform aqueous solutions having a wide range of concentrations. Calculations made with these parameters for a water-reflected cylinder, in which the optimum distribution for minimum mass was approximated within five coaxial regions, agree well with experimental results. Minimum critical masses, concentrations per unit length of cylinder, and masses per unit area calculated for ^{235}U , ^{239}Pu , and ^{233}U are, respectively, 768 g, 16.9 g/cm, 0.417 g/cm²; 492 g, 10.6 g/cm, 0.266 g/cm²; and 571 g, 13.5 g/cm, 0.362 g/cm².

INTRODUCTION

By maintaining adequate limits on only a few variables, fissile material can be kept safely in a subcritical state, regardless of the values of other variables. (Maintaining such limits implies designing equipment, employing instrumentation, and exercising procedural control in such a manner that under no credible contingencies will the limits be exceeded; hence, actual operating limits may have to be more restrictive to allow for such contingencies, the consideration of which is beyond the scope of this paper.) Although the simplicity of ensuring safety in this manner may appeal to the operator of a process, the small values of the limits generally do not; hence, it is usually desirable that the limits be as large as possible, consistent with the requirement that they have adequate assurance of always being subcritical. In aqueous systems where reflection can credibly be limited to that resulting from an effectively infinite thickness of water and where a uniform solution state can be maintained, safety can be ensured solely by a mass or a dimensional limit that can be taken quite close to an experimentally

established minimum critical value. These critical values are known very accurately as the result of extensive critical experiments performed with aqueous solutions of fissile materials¹.

Limits chosen for aqueous solutions are not appropriate where a uniform solution state cannot be maintained, i.e., where precipitates can form or where slurries of metal or of oxide in water are being processed. Not only may the density of fissile material in the slurry be greater than its effective value in solution, thereby decreasing the critical mass or size, but the distribution of fissile material within the slurry may not be uniform. Goertzel² has shown that there is an optimum distribution of fissile material that gives a critical mass lower than that obtainable with a uniform distribution such as usually exists in a solution.

The present paper considers the effect of the distribution of ^{235}U , ^{239}Pu , and ^{233}U in water (at room temperature) on the critical mass in water-reflected spheres, on the critical concentration

¹H. C. PAXTON, J. T. THOMAS, DIXON CALLIHAN and E. B. JOHNSON, "Critical Dimensions of Systems Containing ^{235}U , ^{239}Pu , and ^{233}U ," TID-7028, Los Alamos Scientific Lab. and Oak Ridge National Lab. (1964).

²G. GOERTZEL, "Minimum Critical Mass and Flat Flux," *J. Nucl. Energy*, 2, 193-201 (1956).

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per unit length in water-reflected cylinders, and on the critical concentration per unit area in water-reflected slabs. Five arbitrary symmetrical, monotonically decreasing concentration profiles are chosen (Fig. 1). In the spheres and cylinders these profiles are considered as radial distributions of fissile material, and in the slab, as distributions in the direction perpendicular to the slab faces. The three fissile nuclides are considered to be pure; i.e., ^{234}U , ^{236}U , ^{238}U , and ^{240}Pu are considered to be absent. The uranium isotopes are assumed to have the density of metal in their dispersions in water. The plutonium is assumed to be present as PuO_2 and to have a density of 11.46 g/cm^3 in its dispersion in water. Calculations³ relating to critical experiments have already considered plutonium to be in this form, and the increased density of plutonium in metallic form has too small an effect at the concentrations employed to make adjustment from oxide to metal worthwhile.

METHOD OF CALCULATION

Average critical concentrations of fissile material within water-reflected spheres, cylinders, and slabs having various fixed dimensions were calculated for the concentration profiles of Fig. 1 by two-group diffusion theory with two-group parameters that were normalized to experiments.

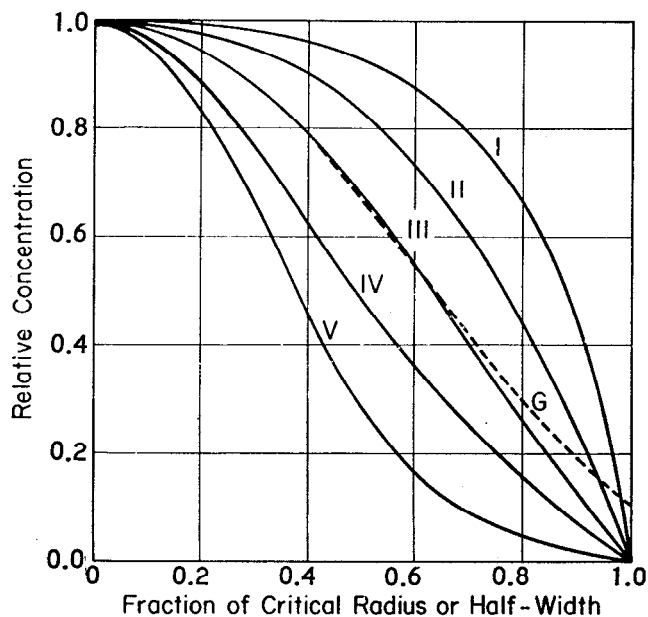


Fig. 1. Concentration distributions.

At low concentrations the logarithm of average critical concentration is a nearly linear function of critical radius of half-thickness; such plots supplemented results obtained in this range. The profiles were approximated by histograms, whose widths conformed to some scheme of subdividing the fissile region, and whose heights were obtained by integrating polynomial fits (Table I) to the profiles over the volumes of the subregions. Except where stated to the contrary, the fissile region was divided into five subregions having equal ratios of surface to volume. The factor by which to multiply relative concentrations given by the histograms to obtain actual critical concentrations within subregions was determined, in each case, by iteration. The two-group calculations made use of tables of values of normalized two-group parameters (at 5-g/liter intervals in the range from 0 to 50 g/liter, at 10-g/liter intervals in the range from 50 to 80 g/liter, and at 20-g/liter intervals for higher concentrations), and five-point Lagrange interpolation was used to obtain parameters at intermediate concentrations.

TABLE I

Coefficients in Polynomial Fits to Concentration Profiles
 $C(r) = 1 + A_2 r^2 + A_4 r^4 + A_6 r^6 + A_8 r^8$

| Profile | A_2 | A_4 | A_6 | A_8 |
|---------|---------|---------|---------|---------|
| I | -0.0692 | -1.2201 | 1.6478 | -1.3585 |
| II | -0.4617 | -1.2143 | 1.3137 | -0.6377 |
| III | -1.2575 | -0.2767 | 0.9909 | -0.4567 |
| IV | -2.9302 | 4.3531 | -3.6156 | 1.1927 |
| V | -4.4472 | 8.2081 | -7.0497 | 2.2888 |

Calculation of Parameters

The form employed for the two-group equations is

$$\frac{1}{3\Sigma_1} \nabla^2 \phi_1 - \Sigma_1 \phi_1 + c_{11} \Sigma_1 \phi_1 + c_{12} \Sigma_2 \phi_2 = 0$$

$$\frac{1}{3\Sigma_2} \nabla^2 \phi_2 - \Sigma_2 \phi_2 + c_{21} \Sigma_1 \phi_1 + c_{22} \Sigma_2 \phi_2 = 0,$$

where subscripts 1 and 2 denote the fast and slow groups, respectively; Σ_i represents the transport cross section; and c_{ji} represents the number of secondary neutrons appearing in group j as the result of an interaction in group i .

For a particular concentration, an asymptotic multigroup transport-theory calculation³ was performed to obtain the material buckling (B^2). The neutron spectrum generated in the calculation was used to calculate average values of Σ_i and c_{ji} . However, since the leakage as computed by the

³H. K. CLARK, "Bucklings of Pu-H₂O Systems," DP-701, Savannah River Lab. (1962).

diffusion approximation used in the two-group equations ($B^2/3\Sigma_i$) is greater than that calculated by the transport expression $[B - \Sigma_i \tan^{-1}(B/\Sigma_i)]/[\tan^{-1}(B/\Sigma_i)]$ used in the multigroup calculations, the source parameters (neutrons per fission and removal cross section, $c_{21}\Sigma_1$) were artificially increased; this was done to maintain, in the two-group calculation, the same material buckling and ratio of fast-to-slow flux components corresponding to this eigenvalue as were obtained in the multigroup calculation⁴.

Adjusted Experimental Sphere Diameters

For ²³⁵U and ²³³U, two-group calculations were made at a series of concentrations for water-reflected spheres of aqueous solutions, having diameters inferred from experiment to be critical, to obtain the buckling increment that must be subtracted to make the sphere critical. (The ²³⁵U diameters were obtained⁵, and the ²³³U diameters were obtained from a similar analysis of experimental data^{6,7}.) Ideally, if calculation and experiment agreed, the buckling increment would be zero. In general, it was not, and a k_{eff} relating experiment and calculation was calculated as

$$k_{eff} = \frac{1 + M^2 B_m^2}{1 + M^2 (B_m^2 - \Delta B^2)},$$

where the migration area (M^2) is that derived from the multigroup asymptotic calculation and ΔB^2 is the buckling increment. Values of k_{eff} so obtained were plotted against the ratio of U:H₂O for both nuclides, and smooth curves were drawn through the points. These curves were then used to obtain critical sphere diameters for uranium metal-water dispersions.

For ²³⁹Pu the critical data for aqueous solutions were complicated by the large number of nitric acid and ²⁴⁰Pu concentrations employed. For these solutions, therefore, the results of calculations³, made directly for the various data points obtained with water-reflected spheres and cylinders and expressed in terms of k_{eff} , were plotted against the concentration of ²³⁹Pu. Since no trends with ²⁴⁰Pu concentration or nitrate concentration were noticed, a smooth curve was drawn through the points.

⁴H. K. CLARK, "Refinements in Two-Group Calculations," *Trans. Am. Nucl. Soc.*, 5, 1, 64-65 (1962).

⁵H. K. CLARK, "Handbook of Nuclear Safety," DP-532, Savannah River Lab. (1961).

⁶J. K. FOX, L. W. GILLEY and E. R. ROHRER, "Critical Mass Studies. Part VIII. Aqueous Solutions of ²³³U," ORNL-2143, Oak Ridge National Lab. (1959).

⁷R. GWIN and D. W. MAGNUSON, "The Measurement of Eta and Other Nuclear Properties of ²³³U and ²³⁵U in Critical Aqueous Solutions," *Nucl. Sci. Eng.*, 12, 364-380 (1962).

For all three isotopes, k_{eff} was found to be quite close to unity at low concentrations (20 to 30 g/liter) and to increase with increasing concentration. The greatest effect was observed with ²³³U solutions, where k_{eff} increased to 1.127 at 200 g/liter. For ²³⁵U solutions the increase was to 1.047 at 480 g/liter, and for ²³⁹Pu solutions, to 1.025 at 200 g/liter. Critical diameters of water-reflected spheres of metal-water dispersions of ²³⁵U and ²³³U were calculated (with two-group parameters for such systems) to be those yielding the values of k_{eff} relating calculation and experiment for U:H₂O ratios that correspond to the concentrations of uranium in water employed in the tables of normalized parameters. These diameters thus represent experimental diameters corrected for the absence of other uranium isotopes, for the absence of oxyfluoride ions associated with uranium in the critical experiments, and for the higher density of uranium metal. Critical diameters of water-reflected spheres of dispersions of PuO₂ in water were similarly calculated with k_{eff} values relating calculation and experiment. The plutonium solutions studied experimentally had been treated as mixtures of PuO₂, N₂O₅, and H₂O, hence, the results applied directly to PuO₂-H₂O dispersions.

At concentrations <25 g/liter for ²³⁵U and ²³⁹Pu, and at concentrations <15 g/liter for ²³³U, the calculations were assumed to be correct, i.e., k_{eff} was taken to be unity. Minimum critical concentrations calculated with this assumption were 12.22, 7.65, and 11.01 g/liter for ²³⁵U, ²³⁹Pu, and ²³³U, respectively, in reasonable agreement with quoted experimental values¹. For water and for concentrations less than the minimum critical values, the multigroup calculations were made with zero leakage, i.e., the spectrum employed to calculate the average two-group parameters was calculated with $B^2 = 0$.

Normalization of Parameters

Two-group calculations were next made with parameters obtained in the multigroup calculations for metal-water or oxide-water dispersions, and with the adjusted experimental water-reflected sphere diameters at the concentrations used in the tables of normalized parameters to obtain the critical buckling increments.

The chief source of error (i.e., of nonzero buckling increments) in these calculations was thought to lie in the fast-group parameter c_{11} . Multigroup calculations of the spectrum were therefore repeated at each concentration with bucklings obtained by subtracting the buckling increment obtained in the two-group calculations from the material buckling originally calculated.

A new set of average Σ_i and c_{ji} was calculated, and c_{11} was then modified to agree with the adjusted buckling. Two-group calculations, made with the adjusted parameters and the adjusted experimental critical diameters, still produced small buckling increments because of a change in extrapolation distance resulting from the change in two-group parameters. The process was therefore repeated until the buckling increments became insignificant. The final set of two-group parameters obtained in this manner was then used to compute critical sizes (Table II). For spheres, the critical diameters were essentially identical with the adjusted experimental diameters, but for slabs and cylinders, because of the change in extrapolation distance introduced by the process of adjustment, these critical dimensions were slightly smaller than those calculated with unadjusted parameters and with values of k_{eff} relating calculation and experiment, the difference increasing with the concentration. For ^{235}U the difference in slab thickness was 0.33 cm, and the difference in cylinder diameter was 0.16 cm at 500 g/liter, the maximum concentration employed. For ^{239}Pu and ^{233}U the differences in slab thickness were, respectively, 0.17 cm and 0.74 cm, and the differences in cylinder diameter were 0.09 cm and 0.36 cm at 200 g/liter, the maximum concentration employed with these nuclides. These differences

were in the conservative direction of predicting smaller critical sizes. Moreover, the concentrations of greatest interest for slabs and cylinders were considerably less than these maximum values, and the differences in critical dimensions, calculated by the two procedures, were correspondingly smaller.

COMPARISON WITH EXPERIMENT

To verify Goertzel's² theory experimentally, Morfitt⁸ performed a pair of critical experiments in an aluminum-walled cylinder subdivided by thin coaxial aluminum partitions into five regions. In one experiment, an approximately optimum distribution of ^{235}U was employed and, in the other, a uniform distribution. The critical height was measured in both cases with the cylinder surrounded laterally by a thick water reflector. There was no top reflector, and the bottom reflection was only that provided by the thick base of the cylinder and the support on which it rested. The dimensions of the regions and the concentrations used for the nonuniform distribution are shown in Table III. The experimental critical

⁸J. W. MORFITT, "Minimum Critical Mass and Uniform Thermal-Neutron Core Flux in an Experimental Reactor," *Nucl. Sci. Tech.*, 4, 107-25, TID-2505 (1954).

TABLE II
Critical Dimensions in Centimeters Calculated with Adjusted Two-Group Parameters for Water-Reflected Spheres, Cylinders, and Slabs Having Uniform Concentrations of ^{235}U , ^{239}Pu , and ^{233}U . For the Sphere and Cylinder, a and b are the Respective Radii. For the Slab, c is the Half-Thickness.

| Concentration g/liter | ^{235}U | | | ^{239}Pu | | | ^{233}U | | |
|--------------------------|------------------|-------|-------|-------------------|-------|-------|------------------|-------|-------|
| | a | b | c | a | b | c | a | b | c |
| 10 | | | | 42.65 | 31.17 | 18.18 | | | |
| 15 | 48.87 | 35.94 | 21.30 | 24.40 | 17.22 | 9.11 | 37.44 | 27.21 | 15.65 |
| 20 | 28.99 | 20.74 | 11.42 | 19.61 | 13.55 | 6.73 | 25.16 | 17.83 | 9.56 |
| 25 | 23.09 | 16.23 | 8.49 | 17.33 | 11.81 | 5.59 | 20.63 | 14.36 | 7.31 |
| 30 | 20.19 | 14.01 | 7.04 | 16.02 | 10.80 | 4.93 | 18.20 | 12.50 | 6.10 |
| 35 | 18.48 | 12.70 | 6.19 | 15.19 | 10.15 | 4.50 | 16.67 | 11.33 | 5.33 |
| 40 | 17.30 | 11.79 | 5.59 | 14.63 | 9.71 | 4.20 | 15.62 | 10.82 | 4.80 |
| 45 | 16.44 | 11.13 | 5.16 | 14.19 | 9.37 | 3.97 | 14.84 | 9.92 | 4.40 |
| 50 | 15.77 | 10.61 | 4.81 | 13.85 | 9.10 | 3.78 | 14.23 | 9.45 | 4.09 |
| 60 | 14.80 | 9.86 | 4.31 | 13.37 | 8.72 | 3.52 | 13.35 | 8.76 | 3.64 |
| 70 | 14.13 | 9.33 | 3.96 | 13.03 | 8.45 | 3.33 | 12.75 | 8.29 | 3.32 |
| 80 | 13.63 | 8.94 | 3.70 | 12.78 | 8.25 | 3.19 | 12.30 | 7.94 | 3.07 |
| 100 | 12.93 | 8.39 | 3.32 | 12.45 | 7.99 | 2.99 | 11.68 | 7.45 | 2.73 |
| 120 | 12.46 | 8.02 | 3.07 | 12.24 | 7.81 | 2.87 | 11.27 | 7.12 | 2.50 |
| 180 | 11.71 | 7.42 | 2.64 | 11.87 | 7.51 | 2.65 | 10.57 | 6.55 | 2.09 |
| 260 | 11.27 | 7.06 | 2.37 | | | | | | |
| 400 | 10.97 | 6.80 | 2.16 | | | | | | |

TABLE III

Dimensions and Concentrations in Cylinder Experiment^a

| Region | I | II | III | IV | V |
|--------------------------|------|------|-------|-------|-------|
| Inner Radius, cm | 0 | 5.54 | 8.14 | 10.27 | 12.55 |
| Outer Radius, cm | 5.48 | 8.08 | 10.21 | 12.49 | 15.10 |
| g ²³⁵ U/liter | 66.8 | 53.8 | 41.3 | 28.5 | 15.16 |

height for the nonuniform loading was 42.7 cm, giving, with the dimensions and concentrations of Table III, a critical mass of 1066 g ²³⁵U. The critical height, concentration, and mass for the uniform loading were, respectively, 41.6 cm, 39.79 g ²³⁵U/liter, and 1162 g.

Two-group calculations were made here with parameters adjusted in the manner described in the preceding section for the UO₂F₂ solution concentrations employed by Morfitt in the experiments. Transmission and escape probabilities were used in calculating the effect of the thin aluminum partitions. The calculations gave a critical axial buckling for the uniform loading of 0.004328 cm⁻² which, together with the experimental critical height, gave a reasonable, average axial extrapolation distance of 3.08 cm. For the nonuniform loading, the critical axial buckling was calculated to be 0.004029 cm⁻² which, with the 3.08-cm extrapolation distance, gave a critical height of 43.34 cm and a critical mass of 1082 g. Thus, if the calculation for the uniform loading were assumed to be correct, then the calculation for the nonuniform loading was in error by 1.5%. The uniform loading would have a *k*_{eff} of 1.0022 (as calculated with *B*² = 0.01764 cm⁻² and *M*² = 29.14), if the height were increased by the difference (0.64 cm) between the calculated and experimental critical heights for the nonuniform loading.

The agreement between experiment and calculation is quite good. Results calculated by the method described in the preceding section therefore should have fairly high accuracy.

RESULTS OF CALCULATIONS

The results of the calculations are presented in the form of graphs of critical mass versus sphere radius (Figs. 2 to 4), critical mass per centimeter of axis versus cylinder radius (Figs. 5 to 7), and critical mass per square centimeter of surface versus slab half-thickness (Figs. 8 to 10). In all cases, the curves for the nonuniform distribution profiles terminate at the curve for the uniform distribution, since masses or concentrations greater than those corresponding to a uniform distribution were not considered to be of interest in the

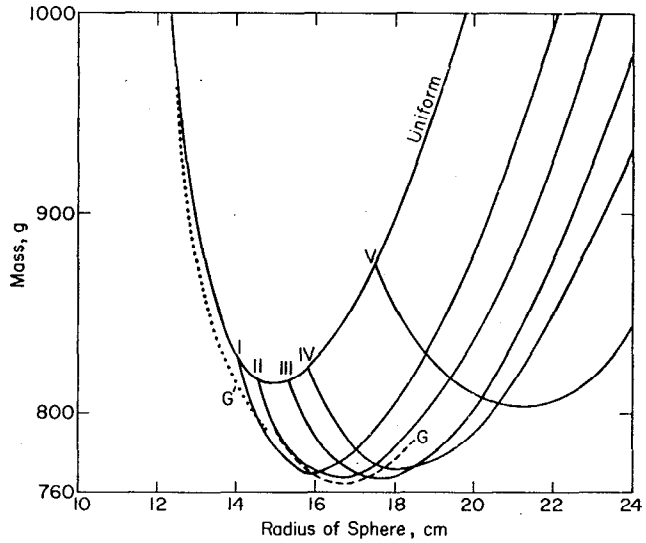


Fig. 2. Critical mass of ²³⁵U in water-reflected sphere as function of distribution.

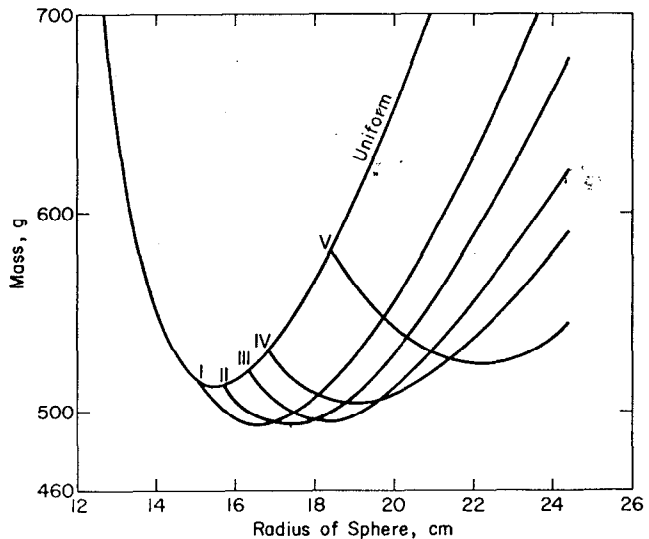


Fig. 3. Critical mass of ²³⁹Pu in water-reflected sphere as function of distribution.

present study. Minima read from the graphs, together with the approximate size, are presented in Tables IV to VI.

Some additional calculations were made for ²³⁵U. Three arbitrary nonsymmetrical concentration profiles were chosen for slabs (Fig. 11) corresponding to

$$A: C(x) = [9 - \exp(2.197x)]/8$$

$$B: C(x) = 1 - x$$

$$C: C(x) = [9 \exp(-2.197x) - 1]/8.$$

The water-reflected slabs were subdivided into four regions of equal volume, and the results

TABLE IV

Minimum Critical Mass as Function of Distribution of Fissile Nuclide in Water

| Distribution | ²³⁵ U | | ²³⁹ Pu | | ²³³ U | |
|--------------|------------------|------------|-------------------|------------|------------------|------------|
| | Mass, g | Radius, cm | Mass, g | Radius, cm | Mass, g | Radius, cm |
| Uniform | 815 | 14.9 | 512 | 15.5 | 598 | 13.6 |
| I | 770 | 15.9 | 492 | 16.6 | 575 | 14.6 |
| II | 768 | 16.7 | 493 | 17.4 | 571 | 15.2 |
| III | 768 | 17.7 | 495 | 18.4 | 572 | 16.0 |
| IV | 772 | 18.1 | 504 | 19.0 | 578 | 16.7 |
| V | 803 | 21.3 | 524 | 22.3 | 600 | 19.3 |
| Goertzel | 764 | 16.7 | | | | |

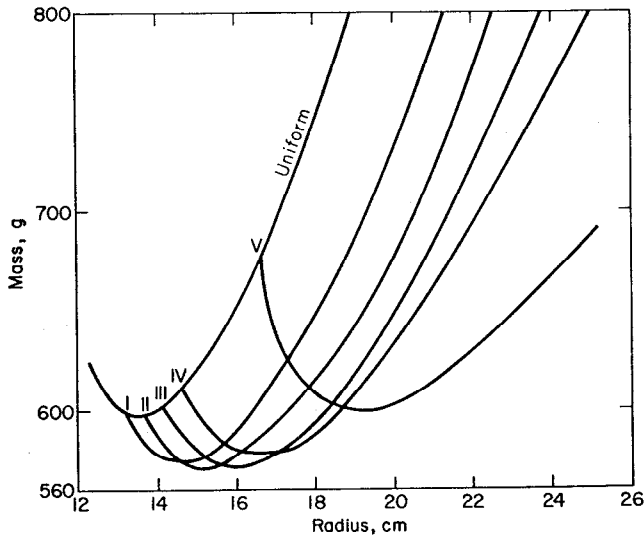


Fig. 4. Critical mass of ²³³U in water-reflected sphere as function of distribution.

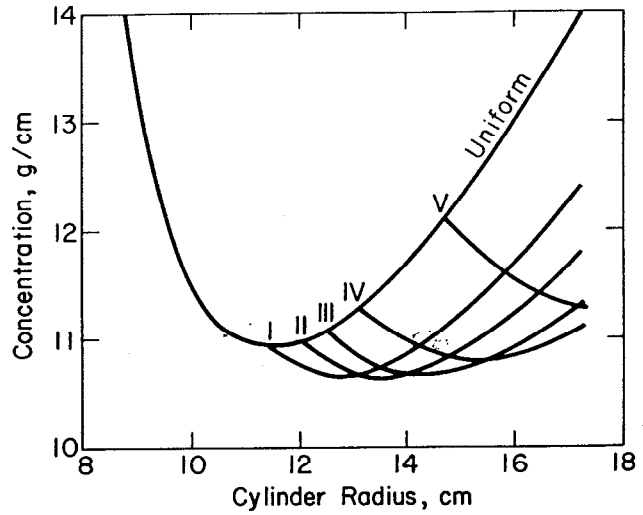


Fig. 6. Critical mass of ²³⁹Pu per unit length.

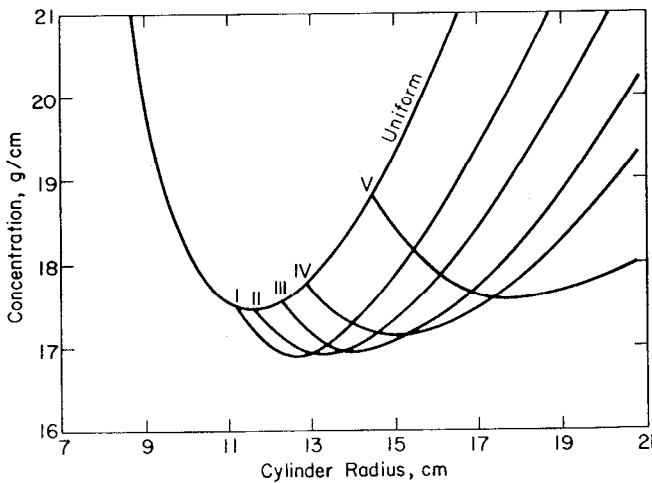


Fig. 5. Critical mass of ²³⁵U per unit length.

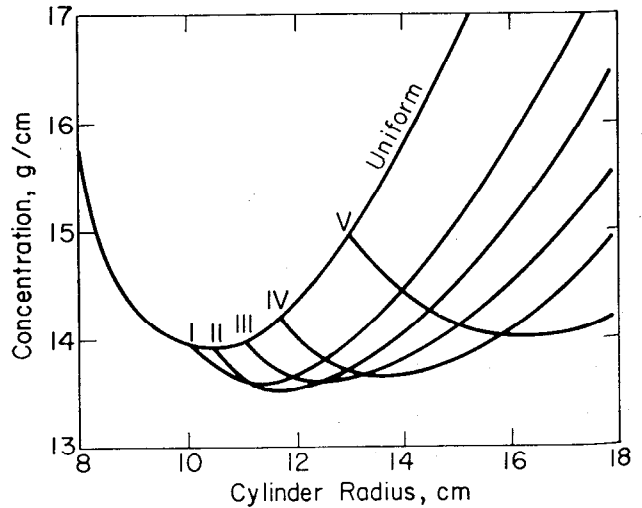


Fig. 7. Critical mass of ²³³U per unit length.

Concentration, g/cm³

Concentration, g/cm²

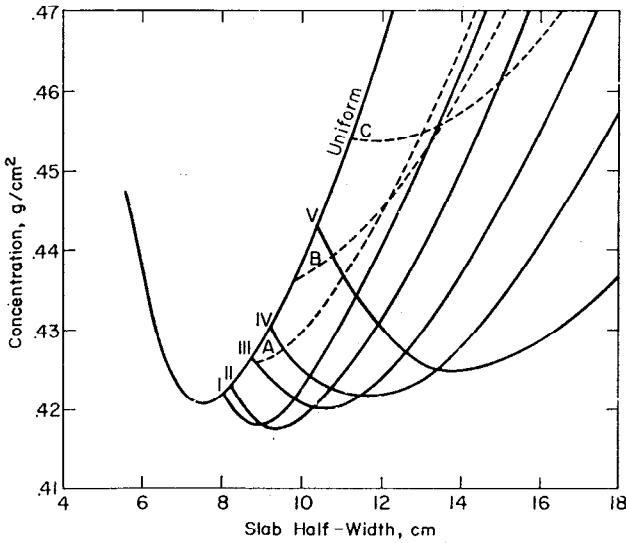


Fig. 8. Critical mass of ²³⁵U per unit area.

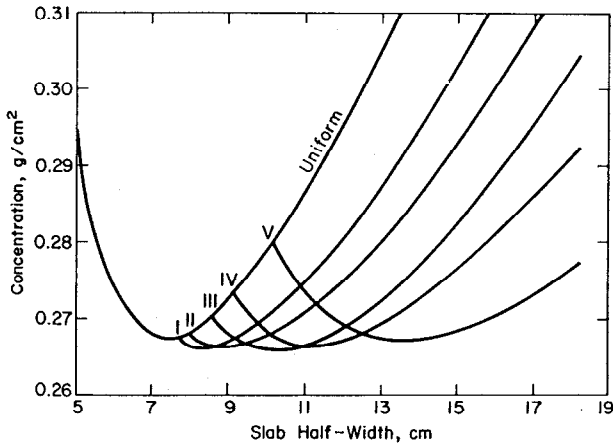


Fig. 9. Critical mass of ²³⁹Pu per unit area.

TABLE V

Minimum Critical Mass per Centimeter of Axis as Function of Distribution of Fissile Nuclide in Water

| Distri- bution | ²³⁵ U | | ²³⁹ Pu | | ²³³ U | |
|-------------------|------------------|---------------|-------------------|---------------|------------------|---------------|
| | Mass, g/cm | Radius, cm | Mass, g/cm | Radius, cm | Mass, g/cm | Radius, cm |
| Uniform | 17.5 | 11.6 | 11.0 | 11.6 | 13.9 | 10.4 |
| I | 16.9 | 12.7 | 10.7 | 12.8 | 13.6 | 11.3 |
| II | 16.9 | 13.2 | 10.6 | 13.5 | 13.5 | 11.8 |
| III | 17.0 | 14.0 | 10.7 | 14.3 | 13.6 | 12.7 |
| IV | 17.1 | 15.1 | 10.8 | 15.4 | 13.7 | 13.7 |
| V | 17.6 | 17.7 | 11.2 | 18.5 | 14.0 | 16.2 |

shown by the dashed curves in Fig. 8 and by Table VI were calculated.

The following polynomial fit was made to the optimum distribution (G) given in Goertzel's² paper:

$$C(r) = 1 - 1.3435 r^2 + 0.1710 r^4 + 0.4070 r^6 - 0.1305 r^8.$$

Calculations were made with this distribution, which was approximated by histograms in the same manner as were the profiles of Fig. 1. The results are shown by the dashed curve in Fig. 2 and in Table IV. The minimum mass calculated here with this distribution is considerably greater than that calculated by Goertzel (690 g at a radius of 16.7 cm).

As stated earlier, all regions in the present study generally had equal ratios of surface to volume. Some calculations were also made in which spheres and cylinders were subdivided into

TABLE VI

Minimum Critical Mass per Square Centimeter of Surface as Function of Distribution of Fissile Nuclide in Water

| Distri- bution | ²³⁵ U | | ²³⁹ Pu | | ²³³ U | |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| | g/cm ² | Half-Width, cm | g/cm ² | Half-Width, cm | g/cm ² | Half-Width, cm |
| Uniform | 0.421 | 7.5 | 0.267 | 7.5 | 0.365 | 6.8 |
| I | 0.418 | 9.0 | 0.266 | 8.4 | 0.362 | 7.8 |
| II | 0.417 | 9.4 | 0.266 | 9.0 | 0.362 | 8.3 |
| III | 0.420 | 10.6 | 0.266 | 10.3 | 0.362 | 9.1 |
| IV | 0.422 | 11.9 | 0.266 | 11.1 | 0.364 | 10.0 |
| V | 0.425 | 13.8 | 0.267 | 13.5 | 0.365 | 12.2 |
| A | 0.426 | 9.0 | | | | |
| B | 0.437 | 9.8 | | | | |
| C | 0.454 | 11.8 | | | | |

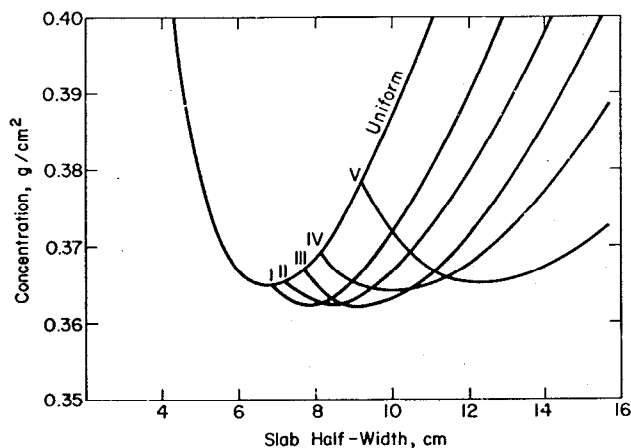


Fig. 10. Critical mass of ^{233}U per unit area.

five regions having equal volume. For the sphere of ^{235}U having the 17.3-cm radius, results obtained with the two schemes of subdivision for profiles I, II, and III agreed within 0.2%, the equal-volume subdivision giving the larger results. For profiles IV and V the equal volume subdivision gave, respectively, masses 0.23% and 3.59% lower than the equal surface-to-volume subdivision. For the sphere having the 28.99-cm radius, the equal-volume subdivision gave masses varying from 0.52% less to 6.38% greater in moving from profiles I through V, and for the sphere having the 14.8-cm radius, profiles I through IV all gave greater (1.3 to 2.4%) masses with the equal-volume subdivision. The agreement between the

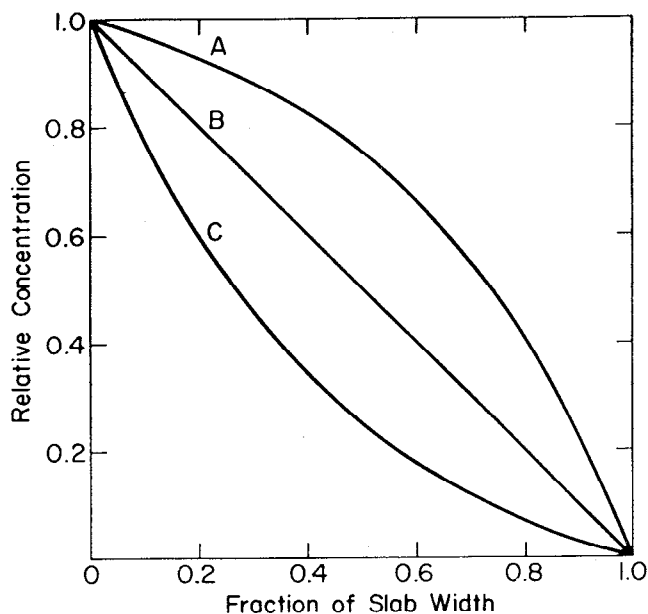


Fig. 11. Nonsymmetrical concentration distributions.

two schemes of subdivision is thus fairly good, particularly for the sizes at which the various profiles have their minimum masses and for which the equal surface-to-volume subdivision gives slightly lower masses. Similar results were observed for ^{239}Pu and ^{233}U spheres. For cylinders, the equal-volume subdivision generally was found to give slightly lower masses per unit length for the cases investigated, but the difference was usually of the order of 0.5% or less.

The good agreement between results obtained with the two schemes of subdivision indicates that more subdivisions would not have produced significantly lower minima. To test this thesis further, however, the 17.3- and 16.44-cm radius spheres of ^{235}U were also subdivided into 2, 3, and 4 regions having equal ratios of surface to volume. (The two-group code used is limited to six regions including the reflector.) Calculations for the former sphere were made with profile III and for the latter with profile G. Results for the former, when plotted against the reciprocal of the number of subdivisions, extrapolated at an infinite number of subdivisions to a mass about 1.3% less than that calculated for five subdivisions. For the latter, the five-subdivision result is 0.1 g greater than the four-subdivision result, and no sensible extrapolation could be made. Extrapolations to an infinite number of subdivisions in the same way with profile III for the ^{235}U cylinder having the 14.01-cm radius and the ^{235}U slab having the 11.42-cm half-thickness indicated, respectively, a mass per unit length and a mass per unit area about 0.5% and 1.0% less than those obtained with five subdivisions. The difference in mass, or concentration, amounts to a difference in k_{eff} of 0.0025 or less.

Examination of Fig. 1 shows that the uniform distribution can be considered as belonging to the same family as profiles I to V. The next member of the family would be formed by superposing a bump on the uniform distribution near $r = 1$. Indeed, Goertzel² has shown that for a restricted radius the optimum distribution is one falling off with increasing r to a nonzero value at the interface with the reflector and, then, having additional fissile material concentrated at the interface. The dotted curve (G') in Fig. 2 was calculated here with the distribution of this type that he presented in his paper. In the present study additional members of the family of profiles were chosen to be flat out to $r = 0.8196$ (the inner boundary of the fifth region of the sphere) and to have linear slopes to concentrations at $r = 1$ that were 1.5 and 2.0 times the value for $0 \leq r \leq 0.8196$. Calculations for ^{235}U spheres, cylinders, and slabs with these profiles (with the core subdivided into two regions, the outer having the same dimensions as

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Goertzel (G as a m at the within lines i Goertzel duces extrem appear approx

Low per unit area tion of is non-tive to though; thermal shapes are no

in the five-region subdivision) were made at average concentrations up to 400 g/liter. The reduction in mass over that of a uniform distribution increased with concentration, and for the cylinder (and even more so for the slab), greater reductions would be achieved by greater relative values at $r=1$. In this range of high concentration, however, mass is probably not so important as size in governing safety. These calculations indicate that, for average concentrations up to about 400 g ^{235}U /liter and hence for masses, masses per unit length, or masses per unit area calculated with this concentration, the critical sphere radius, the critical cylinder radius, and the critical slab half-thickness may be reduced, respectively, by as much as 0.05, 0.1, and 0.15 cm by the concentration of fissile material at the interface with the reflector.

Goertzel² calculated that his optimum distribution (G) would produce a flat thermal flux as well as a minimum mass. Thermal (≤ 0.625 eV) fluxes at the interfaces between regions and at five points within the central region are connected by straight lines in Fig. 12 for several of the ^{235}U spheres. Goertzel's optimum distribution (G) indeed produces a flat flux within the fissile core, but extreme flatness of thermal flux clearly does not appear to be a necessary condition for achieving approximately minimum mass.

CONCLUSIONS

Lower critical masses, lower critical masses per unit length, and lower critical masses per unit area than are attainable with a uniform distribution of fissile material occur when the distribution is nonuniform. The minima are not greatly sensitive to the exact shape of the distribution. Although, as Goertzel shows, a shape giving a flat thermal flux results in a minimum mass, other shapes give masses nearly as small. The minima are not so far below the minimum for a uniform

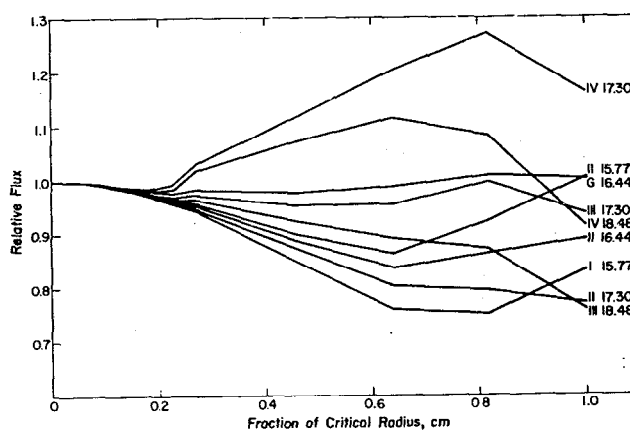


Fig. 12. Thermal flux in ^{235}U spheres.

distribution as Goertzel's calculations indicated. The greatest effect was found for the sphere and for ^{235}U .

Choosing maximum safe parameters is necessarily somewhat subjective and arbitrary. A margin of 0.01 in k_{eff} appears to be ample to allow for uncertainties in the experimental data, and an additional margin of 0.005 appears adequate to allow for errors in the calculations. Maximum values that are safe regardless of distribution, calculated with a migration area of 30 cm^2 and an extrapolation distance of 6.5 cm to correspond to $k_{\text{eff}} = 0.985$, are given in Table VII.

TABLE VII
Maximum Safe Parameters Independent of
Distribution of Fissile Material

| | ^{235}U | ^{239}Pu | ^{233}U |
|------------------------------|------------------|-------------------|------------------|
| Mass, g | 701 | 450 | 525 |
| Mass/length, g/cm | 15.7 | 9.8 | 12.6 |
| Mass/area, g/cm ² | 0.394 | 0.250 | 0.342 |