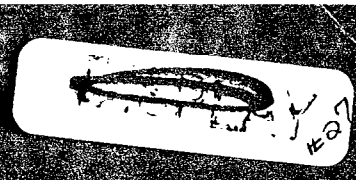

# NUCLEAR SCIENCE and ENGINEERING

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# Dynamic Subcriticality Measurements Using the $^{252}$Cf-Source-Driven Noise Analysis Method

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**Abstract**—*Dynamic measurements of the subcritical neutron multiplication factor $k_{eff}$ using the $^{252}$Cf-source-driven neutron noise analysis method were performed for an unreflected 25.1-cm-i.d. cylindrical tank containing aqueous uranyl nitrate as the solution height was changed at rates of 1 to 23 cm/min, with corresponding changes in $k_{eff}$ from $4 \times 10^{-4}$ to 0.01/s.*

*These experiments, which were the first test of the method to measure $k_{eff}$ while it is changing, showed the following:*

*1. This method has the capability to measure subcriticality for a multiplying system to a $k_{eff}$ as low as 0.30.*

*2. Experimental $k_{eff}$ values can be obtained from the ratio of spectral densities with as little as 6 s of data accumulation and a small fraction of a second analysis time while the solution tank is drained from a height of 29.5 to 6.5 cm in ~60 s, with corresponding changes in $k_{eff}$ from 0.95 to 0.30.*

*3. The measured $k_{eff}$ values obtained do not depend on the speed at which the solution height is changed or whether it is filling or draining.*

*4. The results of the dynamic measurements agreed with the static measurements.*

*5. Where static measurements were practical (limited to $k_{eff}$ down to ~0.5 by detection efficiency) with $^{3}$He proportional counters sensitive to leakage neutrons only, the results agreed with those from measurements with scintillation detectors sensitive to gamma rays and neutrons escaping from the system.*

*6. As in previous experiments, the ratios of spectral densities at low frequency were used successfully to obtain $k_{eff}$ values using a modified point kinetics interpretation of the data.*

*7. The neutron multiplication factors from independent measurements using the break frequency noise analysis method agree with the values of $k_{eff}$ from the measured ratios of spectral densities down to $k_{eff}$ values of 0.65.*

*8. The effectiveness of this method for systems where conditions are changing probably exceeds the dynamic requirements of most nuclear fuel plant processing applications.*

*9. Calculated $k_{eff}$ values using the KENO Monte Carlo code and Hansen-Roach cross sections compare well with the experimental values.*

compressed through four 0.63-cm-diam threaded steel tie rods, equally spaced around the circumference of the vessel, joining the base and a stainless steel upper flange. A 0.63-cm-thick, 32- × 32-cm square acrylic plate on top of the upper flange provided access for the source and minimized evaporation of the solution.

The height of solution above the base plate was determined by an ultrasonic device with a transducer mounted on the acrylic cover of the vessel over a circular opening. Every 0.125 s, this device sampled and stored in the computer the solution height to within ±0.2 mm. Figure 1 is a photograph of the experimental vessel showing the mounted transducer and the $^{252}Cf$ source ionization chamber at the top of the solution.

The vessel was mounted on a 122- × 122-cm square aluminum table 76 cm above a steel grating covered with a stainless steel sheet. This grating was 3.6 m above the concrete floor of the experiment cell. The cell in which the equipment was assembled was ~9.1 × 12.2 × 9.1 m high with thick concrete walls and roof. The experiment vessel was located 4.4 m from the 9.1-m south side of the cell, 3.0 m from the 12.2-m east side, and ~1.85 m from a 2.9-m-diam empty steel tank (2.5 cm thick) also present in the cell. The steel tank

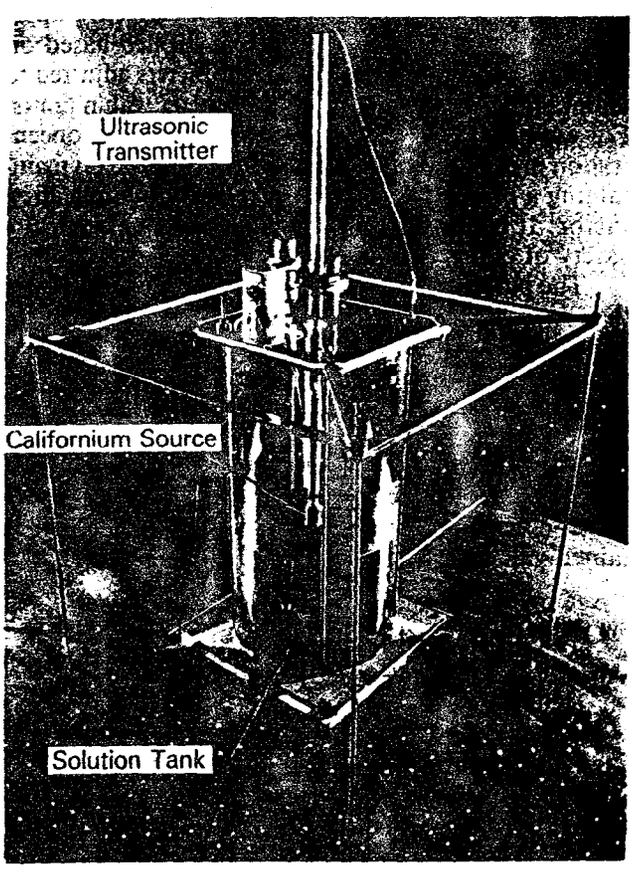



Fig. 1. The experimental vessel.

was located in the cell such that its axis was 4
the 9.1-m south side of the cell and 6.2 m
12.2-m east side.

The aqueous uranyl nitrate contained
and a solution density of 1.643 g/cm$^3$, with
content <0.1 wt% $HNO_3$. Uranium isoto
was $^{234}U = 1.02$, $^{235}U = 93.2$, $^{236}U = 0.41$,
5.37 wt% and was free of significant impu

The $^{252}Cf$ was electroplated on one pla
allel-plate ionization chamber, and the spon
sion rate was 60 000/s (~0.1 μg $^{252}Cf$).
ionization chamber was mounted at the en
cm-o.d., 1.27-cm-i.d. Lexan tube with the
from the chamber inside the Lexan tube (
source was sealed from the solution with sl
and epoxy. The Lexan tubing and the signa
truded out the top of the solution throug
hole in the lid of the experimental vessel.
could be located anywhere along the axis
drical experimental vessel. In most of the
periments, the source was located at the bo
experimental vessel. In practical applicatio
cylindrical tanks, the bottom of the tank
convenient location for the source. For
measurements at various solution height
was located on the axis at the vertical mid
solution, but in some measurements, its
tion on the axis was varied.

Two types of detectors were used in
ments: commercially available $^3He$ propo
ters primarily for static measurements
assembled composite $^6Li$-glass organic sc
both static and dynamic measurements.
portional counters (5.1-cm-diam, 38-cm
Reuter-Stokes model RS-P4-1641-101)
with their axes parallel to the axis of the
vessel adjacent to the outer surface of th
east and west sides, 180 deg apart azin
they detected leakage neutrons. One end
tor was at the bottom of the tank. Thus
tion height changed, the active length o
was essentially the height of the solutio
Two such detectors on the east and we
tank were present for the dynamic mea

The scintillation detectors were c
glass organic scintillators sensitive to fa
neutrons and gamma rays. Each scintill
consisted of a 15.2- × 15.2- × 0.5-cm-
scintillator optically coupled to a 15.2-
cm-thick organic scintillator sensitive t
and gamma rays. The $^6Li$-glass scintill
experimental vessel with the organic sci
it so that the organic was almost neutr
pled from the solution by the absorptic
trons in the $^6Li$. Each detection chann
two adjacent 15.2- × 15.2- × 10.7-cm
tors, each mounted in an aluminum bo
wall) that formed a detection chann

with time is compared with the results of measurements where the solution height was fixed. For these comparisons, the neutron multiplication factor obtained for the average height in the dynamic measurements was compared with static measurements fixed at the average height. The results are also compared with those from break frequency noise analysis measurements.

## STATIC MEASUREMENTS

Static measurements were performed to obtain reference measurements to compare with the results of dynamic measurements. These measurements were performed as a function of solution height with the $^{252}Cf$ source located on the axis at the center of the solution. In the static measurements, the detectors were $^{3}He$ proportional counters 180 deg apart, adjacent to the outer surface of the vessel with their axes parallel to the axis of the experimental vessel and $^{6}Li$-glass orga scintillators located as shown in Fig. 2. The length the $^{3}He$ detectors was more than the height of the lution.

Typical ratios of spectral densities as a function frequency are shown in Fig. 3. These ratios were exa ined visually to determine the range of frequencies low frequency over which the ratio of spectral den ties was constant. This range of frequencies is gen ally greater for more subcritical systems because frequency response of the experimental system $H_s$ has significant amplitude at higher frequency. T range of frequencies over which the ratio was arithm ically averaged was then selected to eliminate high-f quency points for which the statistical uncertainty large and would lead to high or low values of the tio that would distort the average value of the rat Thus, both of these criteria, constant ratio and sm

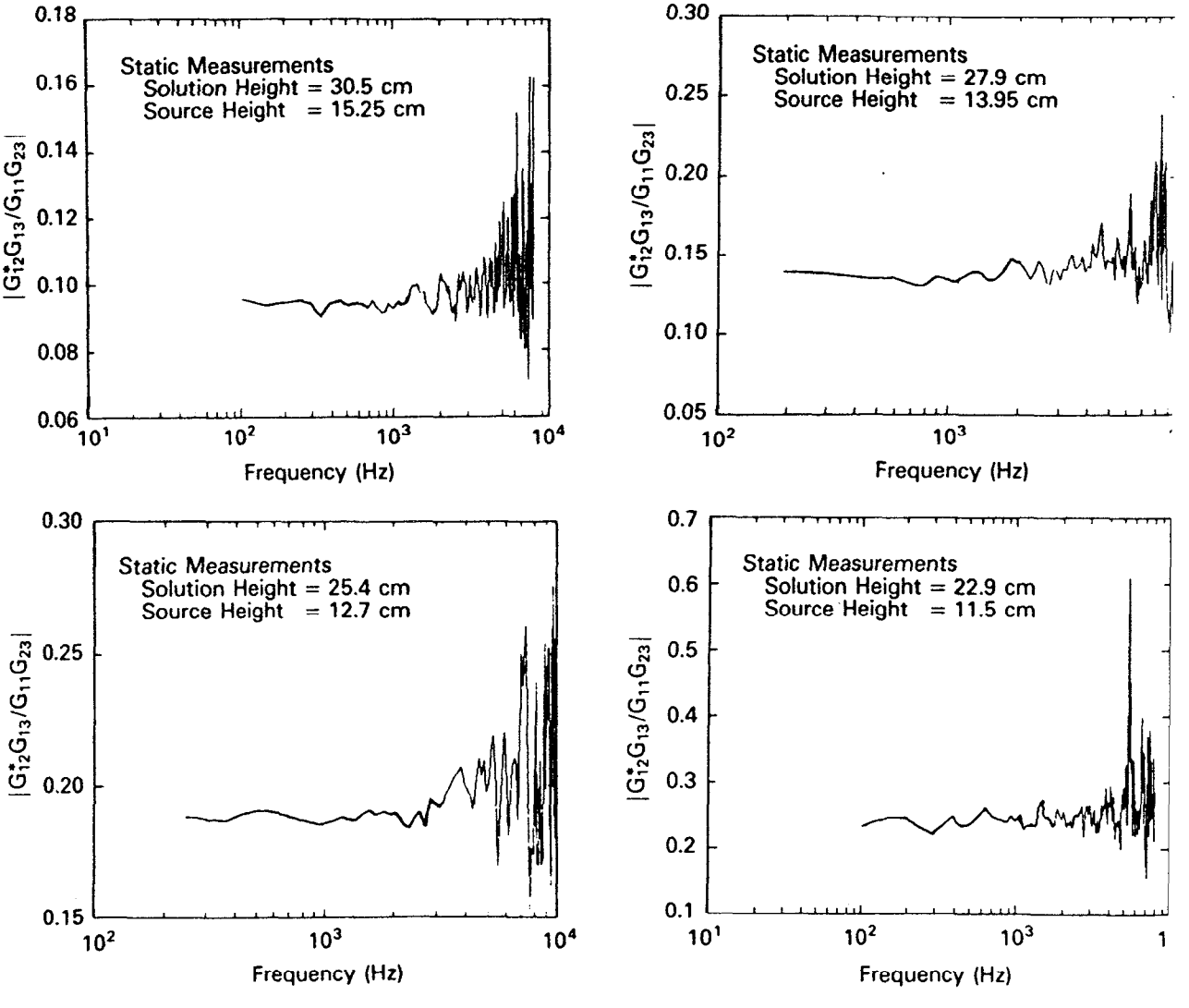



Fig. 3. Ratios of spectral densities as a function of frequency for vari

TABLE I

Ratios of Spectral Densities $G_{12}^{*}G_{13}/G_{11}G_{23}$ at Low Frequency for Various Solution Heights from Static Measureme with Central Source and $^{3}$He Proportional Counters Adjacent to the Outer Surface of the Experimental Vessel

| Solution Height (cm) | Bandwidth of Measurement (kHz) | Number of Data Samples ($10^3$) | Ratios of Spectral Densities[a] ($10^{-3}$) | Upper Limit of Frequency Range for Ratio (kHz) | Neutron Multiplication Factor | |
|---|---|---|---|---|---|---|
| | | | | | Measurement | Calculatio |
| 30.5 | 20 | 80 | 100 ± 1 | 3 | 0.963 | 0.950 |
| 30.5 | 40 | 80 | 96 ± 1 | 3 | 0.964 | --- |
| 27.9 | 40 | 80 | 139 ± 1 | 3 | 0.945 | 0.916 |
| 25.4 | 50 | 240 | 190 ± 1 | 3 | 0.919 | 0.896 |
| 22.9 | 20 | 40 | 248 ± 3 | 5 | 0.884 | 0.870 |
| 20.3 | 100 | 160 | 304 ± 3 | 8 | 0.843 | 0.831 |
| 20.3 | 100 | 40 | 303 ± 5 | 7 | 0.843 | --- |
| 15.2 | 50 | 840 | 465 ± 3 | 5 | 0.704 | 0.721 |
| 10.2 | 100 | 9360 | 525[c] | 0.4[c] | 0.570 | 0.547 |

[a]Uncertainties are one standard deviation of the mean.

[b]Monte Carlo calculations using Hansen-Roach cross sections with the effects of source materials included.

[c]For this measurement, the ratio of spectral densities was not constant at low frequencies (Fig. 3), so the value m sured at 400 Hz was used to obtain the neutron multiplication factor.

were performed using KENO Monte Carlo calculations with Hansen-Roach cross sections (Table C.I in Appendix C provides details of calculations).

Static measurements were also performed with the source at the bottom of the solution with both $^{6}$Li-glass organic scintillators and $^{3}$He detectors. The tios of spectral densities at low frequency for b types of detectors are given in Table II. For low so tion heights (<10 cm), statistically meaningful m surements with the $^{3}$He proportional counters w

TABLE II

Ratios of Spectral Densities at Low Frequency and Neutron Multiplication Factors from Static Measurements with the Source at the Bottom of the Solution

| Solution Height (cm) | Ratio of Spectral Densities[a] | | Neutron Multiplication Factors, $k_{eff}$[b] | | Number of Data Blocks ($10^3$) |
|---|---|---|---|---|---|
| | $^{3}$He Detectors ($10^{-4}$) | Scintillators ($10^{-4}$) | $^{3}$He Detectors | Scintillators | |
| 29.2 | 453 ± 12 (5) | 436 ± 5 (2) | 0.952 ± 0.002 | 0.954 ± 0.001 | 20 |
| 25.8 | 767 ± 40 (5) | 774 ± 20 (2) | 0.924 ± 0.005 | 0.923 ± 0.003 | 20 |
| 20.3 | 1590 ± 20 (5) | 1616 ± 10 (6) | 0.853 ± 0.005 | 0.850 ± 0.005 | 20 |
| 15.3 | 2896 ± 90 (5) | 2843 ± 10 (10) | 0.732 ± 0.015 | 0.738 ± 0.010 | 40 |
| 10.1 | 4508 ± 90 (5) | 4477 ± 60 (10) | 0.535 ± 0.030 | 0.540 ± 0.027 | 20 |
| 9.8 | --- | 4723 ± 60 (10) | --- | 0.505 ± 0.030 | 7 |
| 9.0 | --- | 4960 ± 60 (10) | --- | 0.472 ± 0.035 | 100 |
| 7.5 | --- | 5293 ± 210 (10) | --- | 0.414 ± 0.055 | 40 |
| 6.5 | --- | 5650 ± 60 (10) | --- | 0.346 ± 0.049 | 60 |
| 5.7 | --- | 5943 ± 360 (10) | --- | 0.278 ± 0.097 | 55 |

[a]Values in parentheses are the upper limit of the frequency range in kilohertz over which the ratio was averaged. T precision given is one standard deviation of the mean.

[b]Uncertainties are statistical precision of the ratio of spectral densities and uncertainties in the parameters of App dix B. The largest contribution to the uncertainty was from $I_c/I$ (e.g., 80% of the uncertainty for a solution height 6.5 cm). All tables in this paper other than Tables II and III have uncertainties from the precision of the ratio of spect densities only.

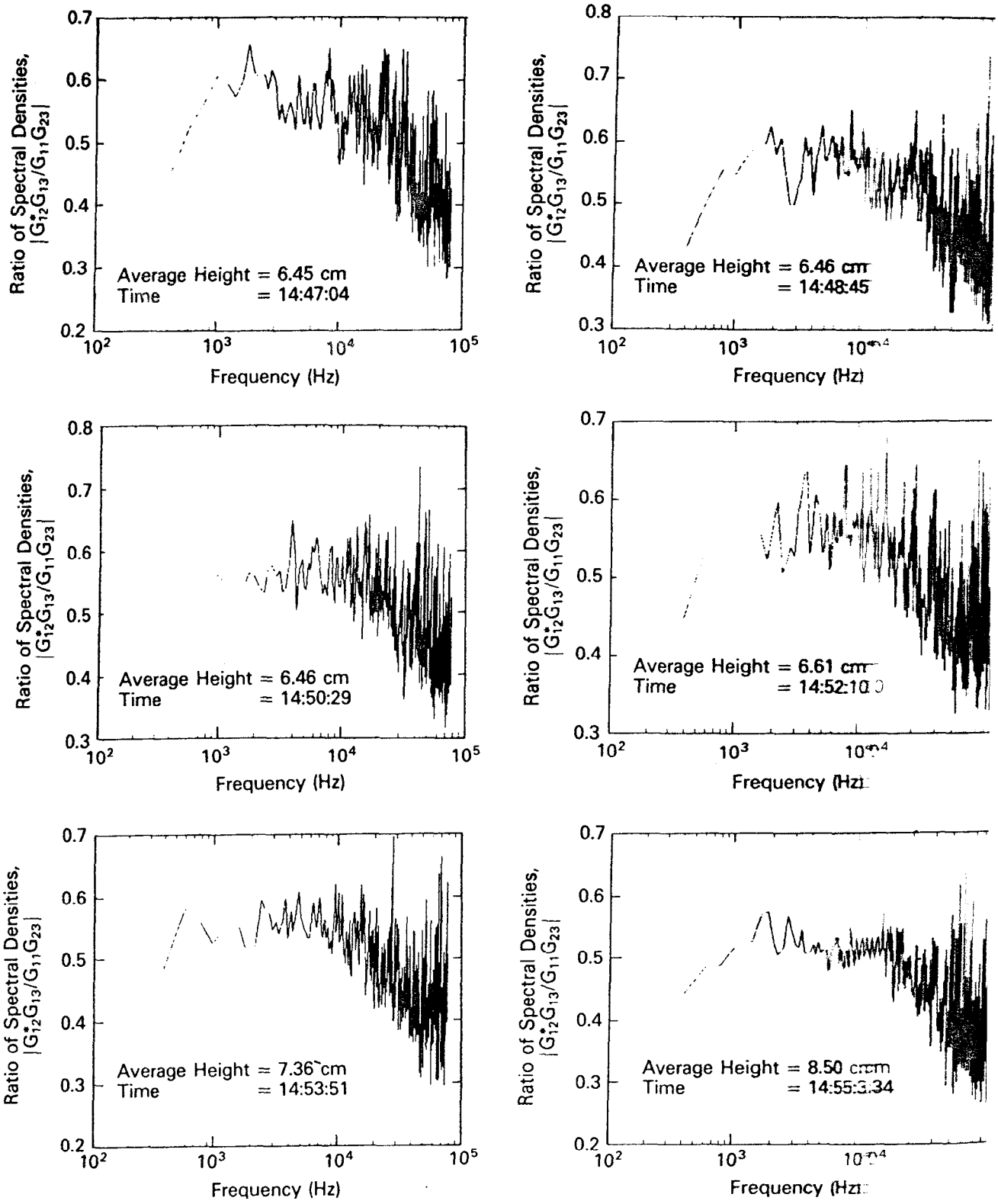



Fig. 5. Ratios of spectral densities as a function of frequency as the experimental vessel was filled at a rate of 1 cm/
(continued on following pages).

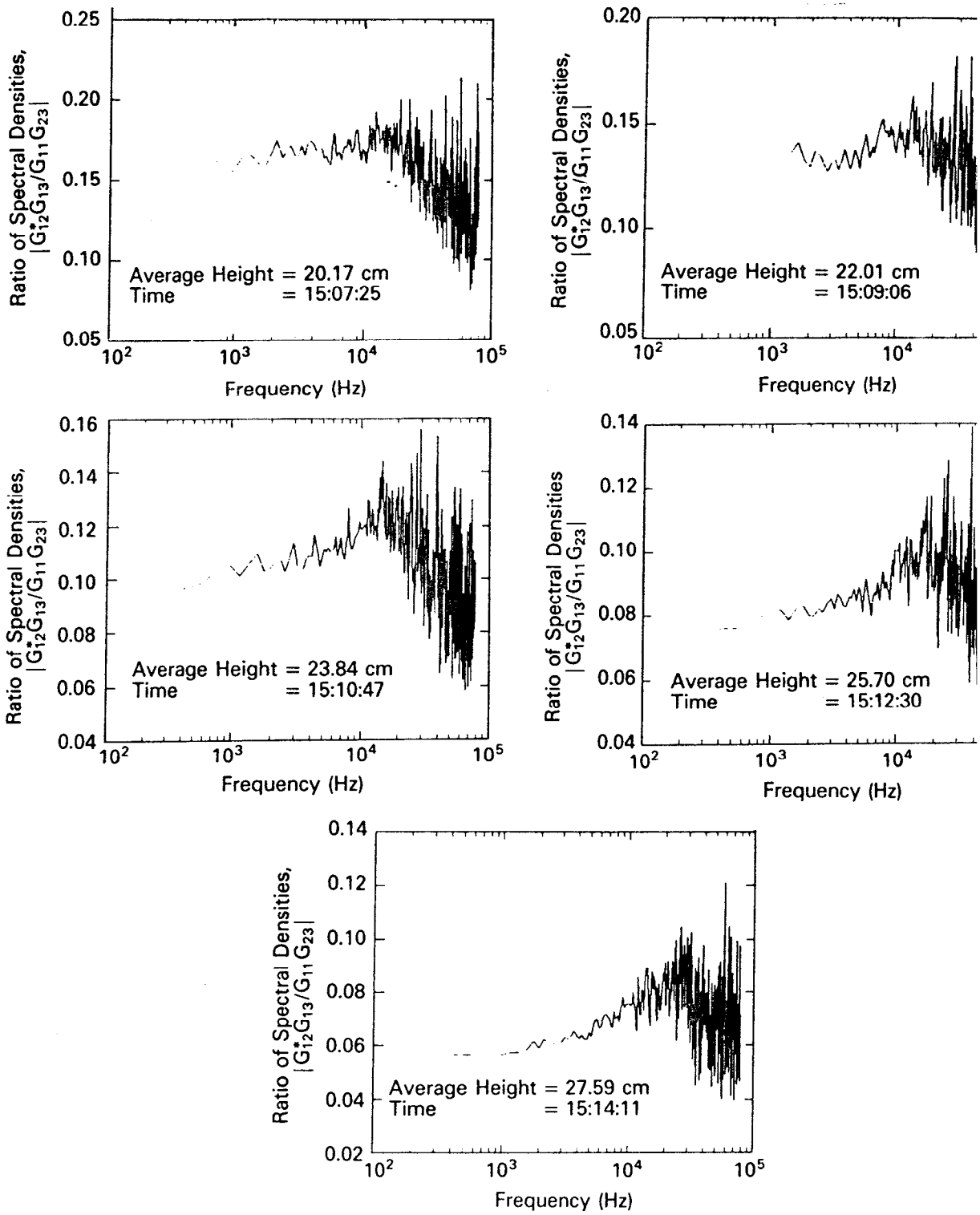
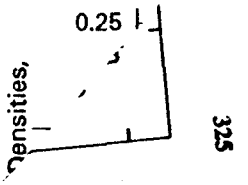



Fig. 5. (Continued)

in the measured neutron multiplication factor varies from 0.01 at the lowest $k_{eff}$ (~0.31) to <0.001 at higher $k_{eff}$ values (0.93).

The first three entries of Table III are for the same solution height. Intercomparison of these thre surements (102 s of data accumulation time) sho the ratios and the resulting neutron multiplicati tors are reproducible even at $k_{eff}$ values as




oherence values and short
herence at low frequency
nnel and the californium
the two scintillator chan-
as a function of solution
; at low frequency at the
m) are $\gamma_{12}^2 = 0.045$ and
olution height of 27.6 cm
As in previous measure-
a function of subcritical-
$\gamma_{23}$.

The experimental vessel was drained at three different rates with the source on the bottom of the tank. The fastest rate was such that the height varied from 29.5 to ~6 cm in ~60 s. This limit on the draining rate resulted from the 1.27-cm-diam opening in the bottom of the tank. Other draining rates were such that the height of solution changed 3 and 5 cm/min.

For a draining rate corresponding to a change of solution height of ~3 cm/min, the data accumulation time before the Fourier processor uploaded the data to the VAX computer for interpretation was ~13 s while that for more rapid draining of the tank (5 and 23 cm/min) was 6.4 s. Thus, these data have a larger statistical error than data for filling the experimental vessel because the data accumulation time is a factor of 8 to 16 shorter. The ratios of spectral densities at low frequency for their draining rates are plotted as a function of solution height in Fig. 6. The $k_{eff}$ values obtained from these measurements are in agreement with those from filling the vessel (Fig. 7).

For a draining rate that corresponds to a change in solution height of 23 cm/min ($\Delta k_{eff}/s \approx 0.01$), the solution was drained continuously from 28.9 to 5.7 cm.

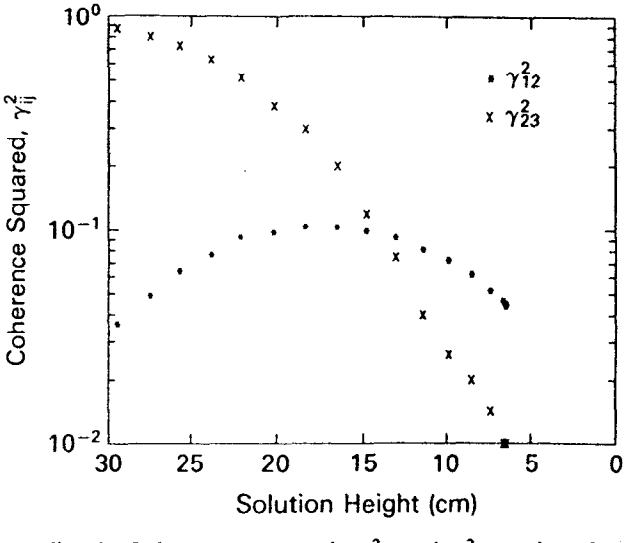



Fig. 8. Coherence squared, $\gamma_{12}^2$ and $\gamma_{23}^2$, at low frequency as a function of solution height for experiments with the californium source at the bottom of the solution and with scintillation detectors.

The draining rate was limited by the size of th
(1.27 cm) provided in the bottom of the tank. T
sults of the data interpretation are given in Tat
The precision of the measured ratio of spectral
ties is such that the precision in the $k_{eff}$ value is (
down to ~0.60 (solution height of 12 cm) for th
collection time of 6.4 s and the draining rate
cm/min. For the last four entries of Table IV, t
lution height was fixed at 5.72 cm and the avera
value obtained is 0.30. The ratios of spectral de
and neutron multiplication factors are plotte
function of solution height in Figs. 5 and 6, wher
are compared with the other dynamic measuren

The rate of change of the neutron multiplic
factor in this experiment was 0.01 in $k_{eff}$ per se
The neutron multiplication factor from all dy
measurements (filling at a rate of 1 cm/min and
ing at rates of 3, 5, and 23 cm/min) and the static
surements are in agreement. Thus, over the ran
filling and draining rates investigated, the $k_{eff}$ v
measured do not depend on the rate of change c
or whether the solution height is increased, decre
or stationary. The dynamic capability of the m
may be more than that required for most nuclea
processing or reprocessing plant applications.

### *Solution Perturbed by Bubbles*

After the experimental vessel was filled to 29.
exploratory experiments were performed with air
bles introduced into the bottom of the tank by le
the pump running with no fuel solution in the
tank. As a result, air was continuously added t
bottom of the tank through a 1.3-cm-diam hole at
dius of 7.6 cm. Air bubbles moved vertically thr
the solution, displacing fuel solution from the int
of the solution tank and also perturbing the upper
face of the solution. This displacement of fuel solu
from the interior to the upper surface decreased th
activity. For this measurement, the source was o
axis at the bottom of the tank.

These exploratory experiments were the first e
iments in which a solution was perturbed during
type of measurement (a perturbed solution tank
be typical of some tanks in in-plant applications). S
these were exploratory measurements, no quantit
characterization of the bubbling was performed o
than the estimate that the volume of air continuo
introduced into the solution was ~500 $cm^3$/min.
data accumulation rates were such that 102 s of
were accumulated by the Fourier analyzer before
data file was uploaded to the VAX computer.

The ratios of spectral densities at low freque
with and without bubbles are given in Table V al
with the $k_{eff}$ values obtained using the same par
eters with and without bubbles. The assumption
the same parameters can be used is probably valid s
the air introduction was not a large perturbation

associated with the bubbles is a small decrease ($\sim 1.6 \times 10^{-3}$).

## BREAK FREQUENCY NOISE ANALYSIS METHOD

The break frequency noise analysis (BFNA) method provides another technique for measuring the subcritical neutron multiplication factor and thus can be compared to the results of the californium-source-driven noise analysis method. It is not completely independent of the californium-source-driven noise method since the BFNA method uses a reference reactivity of $k_{eff} \approx 0.95$ measured by the californium method.

The various spectral densities as a function of frequency can be least-squares fitted to obtain the fundamental mode break frequency $f_b$. The reactivity at a given subcritical state is related to the fundamental mode break frequency at that subcritical state $f_b$ (or to the prompt neutron decay constant $\alpha = 2\pi f_b$) and to the fundamental mode break frequency at delayed criticality $f_{bdc}$ (or prompt neutron decay constant $\alpha_{dc} = \beta/l$) as follows[13]:

$$\frac{\alpha}{\alpha_{dc}} = \frac{f_b}{f_{bdc}} = \frac{1 - k_{eff}}{k_{eff}\beta} + 1 \; .$$

Equation (1) must be corrected for the changes in the neutron lifetime and in the effective delayed neutron fraction from the delayed critical state to the subcritical state of interest as follows:

$$\frac{1 - k_{eff}}{k_{eff}\beta} + 1 = \frac{f_b}{f_{bdc}} \frac{l}{l_{dc}} \frac{\beta_{dc}}{\beta} \; .$$

For the reactivity changes in these experiments corrections for neutron lifetime and effective delayed neutron fraction changes were made. The ratios of prompt neutron lifetimes and the ratios of effective delayed neutron fractions were obtained from calculations of these quantities as a function of solution height, which used fixed source forward fluxes and $k_{eff}$ eigenvalues adjoint fluxes from $S_8$ transport theory calculations.

The break frequency for each solution height was obtained by fitting the various CPSDs and APSDs as functions of frequency $\omega$ to functional forms with (a) a single pole in the transfer function [$H(\omega)$

TABLE VI

Break Frequencies Obtained from Least-Squares Fitting of APSDs and CPSDs and Neutron Multiplication Factors from the BFNA Method as a Function of Solution Height

| Solution Height (cm) | Fundamental Mode Break Frequency ($s^{-1}$) | | Percentage Change in $\frac{l\beta_{dc}}{dc\beta}$ | Reactivity from BFNA ($) | $k_{eff}$ | |
|---|---|---|---|---|---|---|
| | One Mode | Two Mode | | | BFNA | Ratio of Spectral Densities[a] |
| 29.5 | 1369 ± 7[b] | 1328 ± 12 | 0 | 6.13[c] | 0.956[d] ± 0.003 | 0.956[c] ± 0.001 |
| 29.5 | 1385 ± 7 | 1357 ± 24 | 0 | 6.13[c] | 0.956[d] ± 0.003 | 0.956[c] ± 0.001 |
| 29.5 | 1375 ± 7 | 1336 ± 16 | 0 | 6.13[c] | 0.956[d] ± 0.003 | 0.956[c] ± 0.001 |
| 27.6 | 1819 ± 13 | 1776 ± 21 | 1.1 | 8.6 | 0.939 ± 0.004 | 0.940 ± 0.002 |
| 25.7 | 2320 ± 9 | 2265 ± 15 | 2.4 | 11.3 | 0.921 ± 0.004 | 0.921 ± 0.002 |
| 23.8 | 2889 ± 12 | 2850 ± 15 | 3.7 | 14.7 | 0.899 ± 0.005 | 0.896 ± 0.003 |
| 22.0 | 3563 ± 14 | 3503 ± 18 | 5.3 | 18.6 | 0.875 ± 0.006 | 0.872 ± 0.004 |
| 20.2 | 4327 ± 8 | 4308 ± 30 | 6.5 | 23.4 | 0.845 ± 0.007 | 0.845 ± 0.005 |
| 18.4 | 5175 ± 20 | 5117 ± 36 | 9.7 | 28.8 | 0.814 ± 0.009 | 0.812 ± 0.006 |
| 16.5 | 6218 ± 13 | 6280 ± 75 | 13.5 | 36.9 | 0.772 ± 0.010 | 0.771 ± 0.008 |
| 14.7 | 7468 ± 18 | 7760 ± 85 | 18.5 | 47.9 | 0.720 ± 0.011 | 0.723 ± 0.011 |
| 13.1 | 8955 ± 30 | 9629 ± 164 | 24.6 | 62.8 | 0.659 ± 0.013 | 0.662 ± 0.015 |
| 11.4 | 10537 ± 31 | 11196 ± 313 | 32.3 | 77.8 | 0.605 ± 0.009 | 0.588 ± 0.021 |
| 9.9 | 12325 ± 98 | 12849 ± 75 | 44.3 | 97.7 | 0.543 ± 0.014 | 0.522 ± 0.027 |
| 8.5 | 14388 ± 143 | 14825 ± 100 | 56.1 | 122.2 | 0.478 ± 0.014 | 0.443 ± 0.038 |
| 7.3 | 16414 ± 197 | 16750 ± 200 | 71.3 | 152.7 | 0.418 ± 0.014 | 0.376 ± 0.048 |

Note: The change in neutron lifetime for a change in solution height from 29.5 cm to a solution height of 7.3 cm was a factor of 2.1.

[a]Uncertainties are taken from Tables III and VI.
[b]Uncertainties are one standard deviation of the mean from the least-squares fitting of data.
[c]Average of values from Table V unperturbed by air bubbles.
[d]Assumed to be equal to the average value from the ratio of spectral densities.

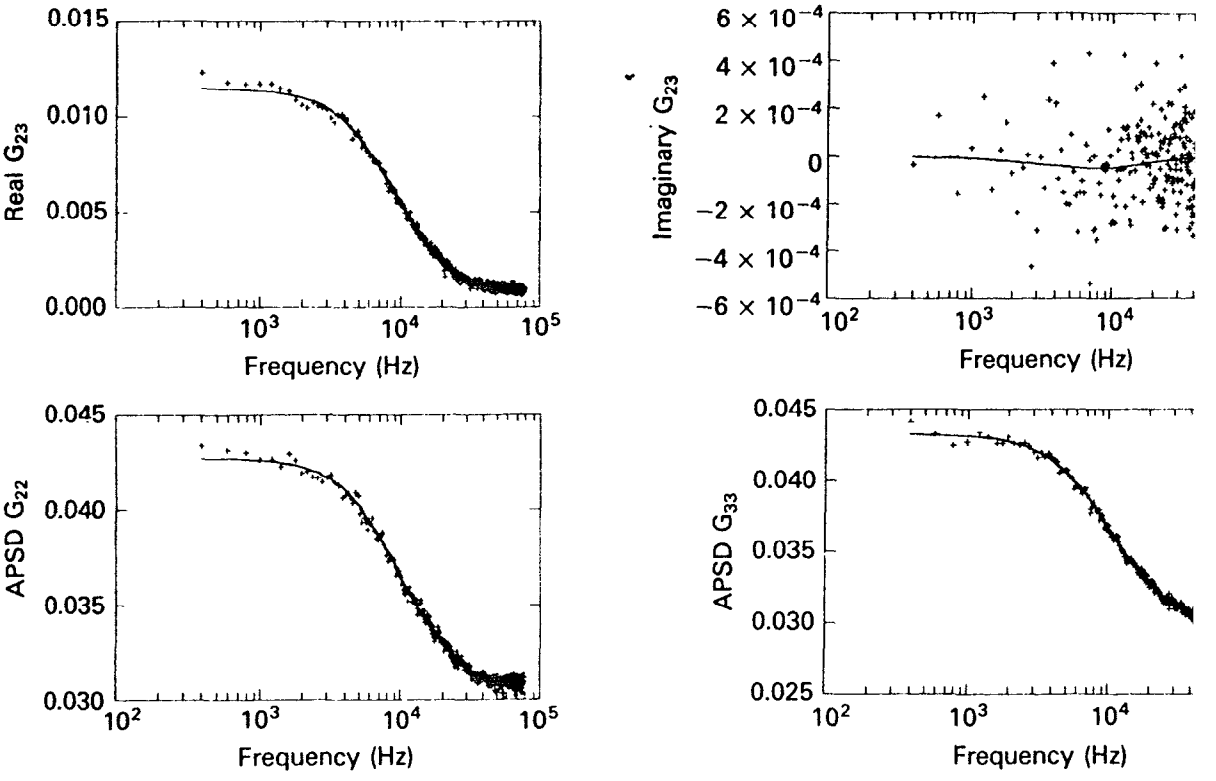



Fig. 9b. CPSD $G_{23}$ and APSDs $G_{23}$ and $G_{33}$ as a function of frequency for a solution height of 13.1 cm v source on the bottom of the tank and the scintillators located as shown in Fig. 2. (Lines are fitted functions.)

fractions, are higher than those from the ratios of spectral densities. In previous experiments, the validity of the BFNA method was usually limited to $k_{eff}$ values down to ~0.8.

## CONCLUSIONS

The results and conclusions of these experiments are as follows:

1. The capability to measure the subcriticality for a multiplying system to $k_{eff}$ as low as 0.3 was demonstrated.

2. Experimental $k_{eff}$ values were obtained from the ratio of spectral densities in times as short as 6 s of data accumulation, and a small fraction of a second analysis time as a solution tank was drained from a height of 29.5 to 6.5 cm in ~60 s with corresponding changes in $k_{eff}$ from 0.95 to 0.30.

3. The measured $k_{eff}$ values obtained did not depend on the speed at which the solution height was changed or whether the tank was filling or draining.

4. The results of the dynamic measurements agreed with the static measurements.

5. Where static measurements were practical (limited to $k_{eff}$ down to ~0.5 by detection efficiency) with $^3$He proportional counters sensitive to neutron the results agreed with those from measuremen scintillation detectors sensitive to gamma rays a trons.

6. As in previous experiments, the ratios o tral densities at low frequency were used succe to obtain $k_{eff}$ values using a modified point k interpretation of the data.

7. The neutron multiplication factors fron pendent measurements using the BFNA metho with the values of $k_{eff}$ from the measured ra spectral densities down to $k_{eff}$ values of 0.65.

8. The effectiveness of this method for s where conditions are changing as demonstrate ably exceeds the dynamic requirements of most fuel plant processing applications.

9. Calculated $k_{eff}$ values using the KENO Carlo code and Hansen-Roach cross sections c well with the experimental values.

The demonstrated dynamic capability method down to neutron multiplication factors allows continuous monitoring of the criticality of process tanks or dissolvers and thus allows r of excess conservatism from plant design, lea

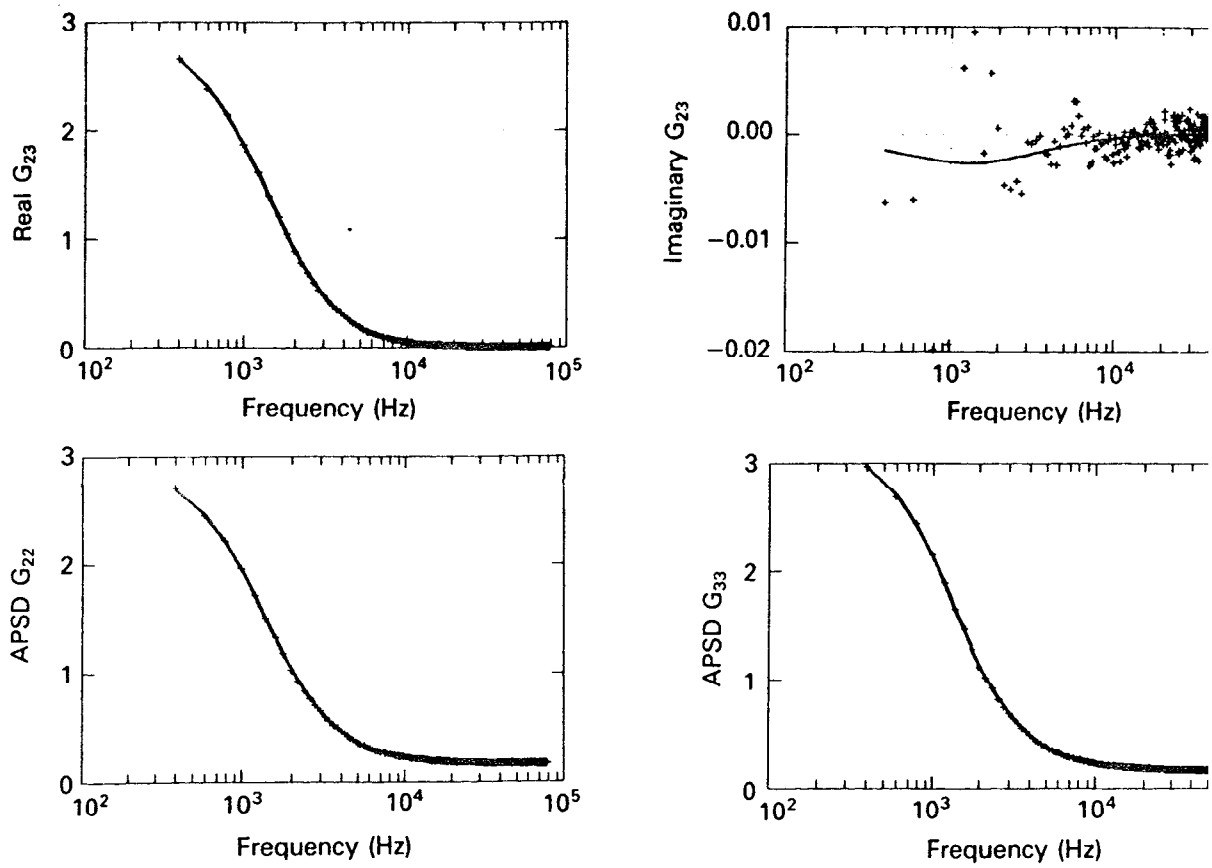



Fig. 9d. CPSD $G_{23}$ and APSDs $G_{22}$ and $G_{33}$ as a function of frequency for a solution height of 29.5 cm v source on the bottom of the tank and the scintillators located as shown in Fig. 2. (Lines are fitted functions.)

for detectors 2 and 3 located outside the fissile material system, as well as the APSD of a detector [Eq. (A.2)], were derived previously using the Langevin equation approach[14] with the assumption that detector noise and reactor noise are not correlated[2]:

$$G_{11}(\omega) = 2|h_1(\omega)|^2\left(\frac{31}{2}F_c\overline{q_\alpha^2} + F_c\overline{q_c^2}\right) , \quad \text{(A.1)}$$

$$G_{22}(\omega) = |h_2(\omega)|^2\left[2W_2F\overline{q_2^2} + \frac{W_2^2(\bar{q}_2)^2}{\bar{\nu}^2}|H_s(\omega)|^2G_s\right] , \quad \text{(A.2)}$$

$$G_{12}(\omega) = 2h_1^*(\omega)h_2(\omega)\bar{q}_c\frac{W_2\bar{q}_2}{\bar{\nu}}H_s(\omega)\frac{\bar{\nu}_cF_cI_c}{I} , \quad \text{(A.3)}$$

and

$$G_{23}(\omega) = h_2^*(\omega)h_3(\omega)\frac{W_2\bar{q}_2}{\bar{\nu}}\frac{W_3\bar{q}_3}{\bar{\nu}}|H_s(\omega)|^2G_s , \quad \text{(A.4)}$$

where

$h_1(\omega), h_2(\omega), h_3(\omega)$ = response of the electronic compon detection systems 1, 2, and 3, tively, at frequency $\omega$

$\bar{q}_c, \bar{q}_\alpha$ = average charge produced in det per $^{252}$Cf spontaneous fission an age charge produced per alpha respectively

$\bar{q}_2, \bar{q}_3$ = average charge produced per inte in detectors 2 and 3, respectively

$W_2, W_3$ = detection efficiency of detection s 2 and 3, expressed as counts per fission

$W_1$ = efficiency for detecting $^{252}$Cf fissi sumed to be $\approx 1$)

$G_s$ = APSD of the reactor noise-equi source

and an asterisk designates complex conjugatio frequencies much larger than delayed neutron constants,

where

$$S_R^{-1} = \frac{\text{multiplication of all neutrons}}{\text{multiplication of californium neutrons}} \frac{1}{A}$$

$$= \frac{Y}{A} \quad \text{(A.8)}$$

or

$$S_R^{-1} = \frac{F_c I_c \bar{\nu}_c + F_i I_i \bar{\nu}_i}{F_c I_c \bar{\nu}_c A} \quad \text{(A.9)}$$

The factor $Y$ accounts for the effect of fissions induced by neutrons from the inherent source contributing to $G_{23}$ but not to $G_{12}$ or $G_{13}$. If not all of the $^{252}$Cf fissions are counted in the detection system electronics, then the fraction of californium fissions counted $A$ must be introduced into $S_R$ to account for the undetected fissions of californium, producing neutrons that induce fissions in the system that contribute to $G_{23}$ but not to $G_{12}$ and $G_{13}$.

When a multiplying system is far subcritical, the power spectral density of the noise-equivalent source $G_s$ must be modified to include $X'$, a modified form of the neutron dispersion number given by Eqs. (A.5) and (A.6). Then, $(1 - k_{eff})/k_{eff}$ is of the form

$$\frac{1 - k_{eff}}{k_{eff}} = \frac{C_1 G_{12}^* G_{13}/G_{11} G_{23}}{1 - C_2 G_{12}^* G_{13}/G_{11} G_{23}} \quad \text{(A.10)}$$

instead of

$$\frac{1 - k_{eff}}{k_{eff}} = C \frac{G_{12}^* G_{13}}{G_{11} G_{23}} , \quad \text{(A.11)}$$

where $C$, $C_1$, and $C_2$ are constants involving g parameters defined by Eqs. (A.1) through (A.11), some of which depend on solution height.

If it is assumed that reactor and detector noises are completely correlated,[16,17] the value of $C_2$ is $\overline{\nu_c(\nu_c - 1)}/(\bar{\nu}_c)^2$ while the value of $C_1$ is essentially the same. The sensitivity of the interpretation of the data to the assumption of complete correlation between detector and reactor noises is examined in Appendix B.

If spatial modal effects are significant, the ratio of spectral densities can be multiplied by a calculated spatial modal correction factor $M_c$, which is the ratio of $G_{12}^* G_{13}/G_{11} G_{23}$, obtained from fundamental mode only, to that obtained with all modes present. Since there is a modal correction factor $M_{ij}$ for each CPSD $G_{ij}$, point kinetics interpretation of the data can yield valid results if the modal correction to the numerator is the same as that for the denominator (i.e., $M_{12} M_{13}/M_{23} = 1$). This factor can be calculated according to the methods of Verdu-Martin et al.[18] and Verdu-Martin[19] for unreflected cylindrical geometries. An alternate approach[20,21] is to fit the data to sums of modes if practical and obtain the fundamental mode ratio directly from the experimental data. In general, this separation of the fundamental mode ratio is not practical.

## APPENDIX B

## PARAMETERS FOR INTERPRETATION OF TI RATIO OF SPECTRAL DENSITIES TO OBTAI THE NEUTRON MULTIPLICATION FACTOI AND SENSITIVITY STUDIES

Several parameters that appear in Eqs. ( through (A.11) in Appendix A and are used for i pretation of the ratio of spectral densities to obtai neutron multiplication factor can be independe measured or obtained from measurements of ot The origins of these parameters and their values ar scribed below. Also, the sensitivity of the interp tions to variations in these parameters is presente a function of $k_{eff}$ value.

The value of $q_\alpha$ is zero. This results from the tronics associated with the $^{252}$Cf ionization cham which discriminates against the contribution to the nal from alpha-particle decay in the ionization ch ber. The value of $q_c$ is unity because the contribut to the signal from all $^{252}$Cf fissions are the same

Since there is no significant inherent neu source in the uranyl nitrate solution comparable t size of the $^{252}$Cf source, the third term in Eq. (A. zero, and since >99.5% of the $^{252}$Cf spontaneous sion produces signals in channel 1 and these signal counted, $Y = A = S_R = 1$.

The average number of neutrons per uranium sion was obtained by using the fluxes from trans theory calculations and the ENDF/B-IV data[2] calculate the total number of neutrons per uranium sion. This averaging accounts properly for the de dence of the average number of neutrons per fissio the energy of the neutron inducing the fission. Ch ing solution height resulted in a <0.2% change in value of the number of neutrons per fission, and resulting average number of neutrons per uranium sion was 2.426. The value of the average numbe prompt neutrons per fission $\bar{\nu}$ is 2.408 and was used interpretation at all solution heights.

The number of delayed neutrons per uranium sion was calculated from the delayed neutron yi the delayed neutron effectiveness factors obtained ing the forward and adjoint fluxes from transport ory,[23] and the delayed neutron spectra of Batch and McKhyder.[24] The effective delayed neutron f tion calculated in this way is 0.0075.

The value of the Diven factor [$X = \overline{\nu(\nu - 1)}$ was obtained from the measurements of prompt tron multiplicities for uranium. Its value does no pend significantly on the energy of the neu inducing fission and varies only from 0.796 to 0. when the neutron energy varies from thermal to 2 (Ref. 25). The value of $X$ used was 0.80. The valu the number of prompt neutrons per $^{252}$Cf fission its mean square ($\bar{\nu}_c$ and $\overline{\nu_c^2}$) are from the meas ments of prompt neutron multiplicities of Spe et al.,[26] $\bar{\nu}_c = 3.773$ and $\overline{\nu_c^2} = 15.818$.

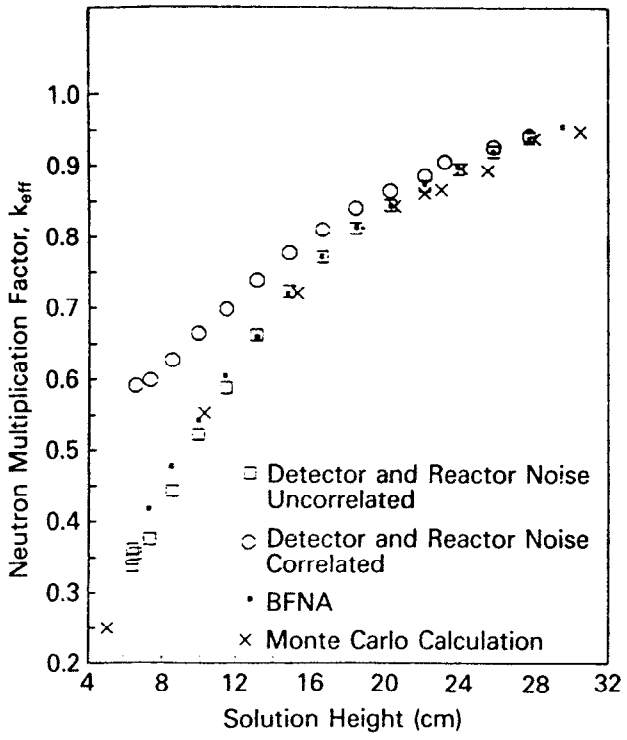



Fig. B1. Sensitivity of neutron multiplication factors to assumptions concerning correlation of reactor and detector noises.

yields $k_{eff}$ values that differ from those for the un related assumption below 0.85, and the differenc creases as $k_{eff}$ decreases to as much as a factor of at $k_{eff} = 0.35$. The results of the interpretation, suming that reactor and detector noise are correla also differ from other measurements and calculati

## APPENDIX C

## CALCULATED NEUTRON MULTIPLICATION FACTORS

The neutron multiplication factors were calcul using the Monte Carlo method with the KENO co and Hansen-Roach[29] and ENDF/B-IV cross section The calculations included the source and its Lexan t and the reflection effect of the scintillation detect as located in the measurements. The results of th calculations are given in Table C.I. The small reacti effect of the detectors that cannot be reliably obtai from the calculations by subtracting the calculated n tron multiplication factors from calculations with without the detector was included because of the tistical uncertainty of the calculations (±0.005). $k_{eff}$ values using the ENDF/B-IV data were con tently higher than those from Hansen-Roach cross tions by an average of 2% in $k_{eff}$. The calculated

TABLE C.I

Neutron Multiplication Factors from Monte Carlo Calculations with Hansen-Roach Cross Sections

| Solution Height (cm) | Calculated Neutron Multiplication Factors[a] | | | |
|---|---|---|---|---|
| | Hansen-Roach Cross Sections | | | ENDF/B-IV 27 Group (No Source or Scintillators[d]) |
| | Source and Scintillators[b] | Source Only[c] | No Source or Scintillators[d] | |
| 30.5 | 0.949 | 0.950 | 0.963 | 0.982 |
| 27.9 | 0.939 | 0.916 | --- | --- |
| 25.4 | 0.894 | 0.896 | 0.902 | 0.938 |
| 24.0 | 0.897 | --- | --- | --- |
| 22.9 | 0.867 | 0.870 | --- | --- |
| 22.0 | 0.862 | --- | --- | --- |
| 20.4 | 0.844 | 0.831 | 0.841 | 0.868 |
| 15.2 | 0.721 | 0.721 | 0.741 | 0.763 |
| 10.2 | 0.553 | 0.547 | 0.544 | 0.574 |
| 5.08 | --- | 0.241 | 0.249 | 0.256 |

[a]Statistical error is ~0.005. All calculations included the bottom, sides, and top of the tank. Where the source w included in the calculation, the 2.54-cm-o.d. Lexan tube was included in the calculation. As a result, with the source locat at any height of solution, the Lexan tube displaced solution on the axis of the cylinder above the source position.

[b]Source located at the bottom of the tank and the scintillators as shown in Fig. 2 were included in the calculatio

[c]Source at the center of solution height on the axis of the tank was included in the calculation.

[d]Source and scintillators were not included in the calculations.

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## REFERENCES

1. J. T. MIHALCZO, V. K. PARÉ, G. L. RAGAN, M. V. MATHIS, and G. C. TILLETT, *Nucl. Sci. Eng.*, **66**, 29 (1978).

2. J. T. MIHALCZO, W. T. KING, and E. D. BLAKEMAN, "$^{252}Cf$-Source-Driven Neutron Noise Analysis Method," presented at Workshop on Subcriticality Reactivity Measurements, Albuquerque, New Mexico, August 1985, CONF-8508105.

3. Memorandum of Agreement between the U.S. Department of Energy and the Power Reactor and Nuclear Fuel Development Corporation, Japan, in the area of Criticality Data Development, signed August 12, 1983.

4. W. T. KING, J. T. MIHALCZO, and E. D. BLAKEMAN, *Trans. Am. Nucl. Soc.*, **47**, 239 (1984).

5. J. T. MIHALCZO and W. T. KING, *Trans. Am. Nucl. Soc.*, **43**, 408 (1982).

6. J. T. MIHALCZO and W. T. KING, *Nucl. Technol.*, **84**, 205 (1989).

7. J. T. MIHALCZO, W. T. KING, and E. D. BLAKEMAN, *Trans. Am. Nucl. Soc.*, **49**, 241 (1985).

8. J. T. MIHALCZO, E. D. BLAKEMAN, and W. T. KING, *Trans. Am. Nucl. Soc.*, **52**, 640 (1986).

9. J. T. MIHALCZO, W. T. KING, and E. D. BLAKEMAN, *Trans. Am. Nucl. Soc.*, **50**, 307 (1985).

10. W. T. KING and J. T. MIHALCZO, *Trans. Am. Nucl. Soc.*, **33**, 796 (1979).

11. J. T. MIHALCZO, W. T. KING, E. B. JOHNSON, and E. D. BLAKEMAN, *Trans. Am. Nucl. Soc.*, **45**, 337 (1983).

12. M. V. MATHIS, J. T. DeLORENZO, M. M. CHILES, and J. T. MIHALCZO, "Nuclear Detection Instrumentation for Reactivity Measurements with the Fast Flux Test Facility Engineering Mockup Core," *IEEE Trans. Nucl. Sci.*, NS-22, 691 (1975).

13. C. W. RICKER, D. N. FRY, E. R. MAN HANAUER, "Investigation of f Negative Rea surement by Neutron Fluctuation Analysis," *Noise Analysis in Nuclear Systems*, Gainesvi November 1963.

14. P. LANGEVIN, *Acad. Sci. Paris CR*, **46**

15. C. E. COHN, *Nucl. Sci. Eng.*, **7**, 472 (19

16. Y. YAMANE, S. WATANABE, K. NI MIYOSHI, T. SUZAKI, and I. KOBAYASHI *ergy Soc. Jpn.*, **28**, *9*, 850 (198636).

17. F. C. DIFILIPPO, *Nucl. Sci. Eng.*, **99**, 2

18. G. VERDU-MARTIN, J. L. MUÑOZ-C MIHALCZO, and W. T. KING, *Trans. Am.* **46**, 452 (1984).

19. G. VERDU-MARTIN, "Teoroia General de Estocastico de Neutrones y su Aplicacion a la Reactividad en Conjuntos Subcriticos," Thesis, Politecnica de Valencia (Apr. 1984).

20. C. MARCH-LEUBA, "Interpretation of S Measurement with Strong Spatial Effects," Thes versity of Tennessee (1987).

21. C. MARCH-LEUBA, J. MARCH-LEUBA DIFILIPPO, *Trans. Am. Nucl. Soc.*, **54**, 348

22. G. D. GARBER, "ENDF-201, ENDF/B Documentation," BNL-17541, Brookhaven Nat ratory (1975).

23. W. A. RHOADES, D. B. SIMPSON, R. L and W. W. ENGLE, Jr., "The DOT-IV Two-D Discrete Ordinates Transport Code with Space Mesh and Quadrature," ORNL/TM-6529, Oak tional Laboratory (1979).

24. R. BATCHELOR and H. R. McKHYDE *Energy*, **3**, 7 (1956).

25. M. S. ZUCKER and N. E. HOLDEN, *Nucl. Soc.*, **52**, 636 (1986).

26. R. R. SPENCER, R. GWIN, and R. INC *Sci. Eng.*, **80**, 603 (1982).

27. L. M. PETRIE and N. F. CROSS, "KENO- proved Monte Carlo Criticality Program with S ing," ORNL/NUREG/CSD-2, Vol. 2, Oak Ridg Laboratory (1985).

28. L. M. PETRIE and N. F. CROSS, "KENO- proved Monte Carlo Criticality Program," O Oak Ridge National Laboratory (1975).

29. G. E. HANSEN and W. H. ROACH, "Six a Group Cross Sections for Fast and Intermediate C semblies," LAMS-2543, Los Alamos Scientific L (1961).