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Material Buckling and Critical Masses of Uranium Rods Containing 3 wt %U^{235} in H_2O*

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Exponential experiments with light-water moderator were conducted to determine criticality standards for the handling of uranium metal enriched to 3 wt % U^{235}. These measurements, made with massive rods 2 and 3 in. in diameter, were combined with Hanford measurements with smaller rods to provide critical bucklings and masses for H_2O-moderated lattices over a range of rod diameters from less than 0.15 in. to more than 3 in. Subcritical buckling measurements are compared with the more conventional approach-to-critical method.

INTRODUCTION

Experiments were conducted at the Savannah River Laboratory (SRL) and Hanford Laboratories[1] to establish standards for the prevention of accidental criticality in handling, storing and processing uranium metal enriched to 3 wt % U^{235}. These experiments determined material bucklings and critical masses over a wide range of H_2O-moderated rod lattices; the Hanford studies were on rods with diameters from 0.175 to 0.925 in., while the SRL studies were on rods with diameters of 2 and 3 in. Taken together, these measurements form a reasonably complete set of experimental data for extrapolation or interpolation to any practical rod diameters.

Material for the SRL experiments, as well as much of the impetus behind the experiments, were furnished by the Raw Materials Feed Center at Fernald, Ohio.

In the SRL experiments, subcritical exponential techniques were used to determine the material bucklings of the test lattices. The critical mass can then be calculated for any given shape using the measured material buckling. In the Hanford measurements, the approach-to-critical method was used to measure critical mass directly. In addition, buckling measurements were made on all but one rod size.

Results from the exponential experiments and from the approach-to-critical experiments can be analyzed to yield equivalent data. The subcritical method has the twin advantages of being inherently safer and of offering applicability to any proposed shape.

The exponential method takes advantage of the fact that the material buckling for a given lattice arrangement is independent of the shape of the lattice. Hence, once the material buckling of a lattice is determined, the critically safe size of this lattice, or the critically safe number of fuel pieces, may be calculated providing only that the geometrical buckling of the proposed shape is calculable. Any lattice for which the material buckling exceeds the geometrical buckling has an effective neutron-multiplication constant greater than unity and is 'unsafe'. Thus in terms of the one-velocity diffusion approximation, the effective neutron multiplication constant can be expressed as

$$k_{eff} = \frac{1 + M^2 B_m^{\ 2}}{1 + M^2 B_g^{\ 2}}$$

where M^2 is the effective neutron diffusion area. The only assumption is that the extrapolation distance is not dependent on the lattice shape.

*The information contained in this article was developed during the course of work under contract AT(07-2)-1 with the USAEC.

[1]R. C. LLOYD, "Summary Listing of Subcritical Measurements of Heterogeneous Water Uranium Lattices Made at Hanford," HW-65552, pp. 40-49 (June 8, 1960).

Approach-to-critical measurements, on the other hand, determine directly the critical number of rods at a given lattice pitch for a given geometrical shape. Unless these data are used to infer a material buckling by equating critical and material bucklings at $k_{eff} = 1$, the measurement has to be repeated for any change in lattice shape.

SRL EXPERIMENTS

The material-buckling measurements at SRL were made in an exponential facility that consisted of a cylindrical aluminum tank 5 ft in diameter and 7 ft high. This tank rests on top of, and is supplied with neutrons from, a small reactor fueled with enriched uranium and moderated with graphite. The test fuel pieces, which were unclad rods 4 ft in length, were suspended from the top of the exponential tank. An effectively infinite H_2O reflector surrounded all test lattices. The material-buckling determinations consisted of measuring the vertical flux distribution by pin activations and fitting this distribution to the function

$$\phi(Z) = A \sinh K_z(t - Z) \tag{1}$$

where

A = constant depending on neutron density
K_z = (neutron relaxation length)$^{-1}$, in the vertical direction
t = extrapolated height of lattice, measured from the bottom of the lattice
Z = distance in a line parallel to the rods, measured from the same base plane from which t is measured.

The material buckling, B_m^2, is then given by

$$B_m^2 = B_x^2 + B_y^2 - K_z^2 \tag{2}$$

where B_x^2 and B_y^2 are the transverse geometrical bucklings in the x and y directions, perpendicular to the z direction. The transverse bucklings can be expressed as

$$B_x^2 \text{ (or } B_y^2) = \left(\frac{\pi}{NP + 2\lambda}\right)^2 , \tag{3}$$

where

N = the number of unit cells in the x (or y) direction,
P = the lattice pitch, and
λ = the extrapolation distance, here defined as the distance beyond the unit cell boundary where the flux extrapolates to zero.

The extrapolation distance was treated as an unknown because it is a complex function of rod diameter, rod enrichment, and lattice pitch. Since (2) contains two unknowns, B_m^2 and λ, it was necessary to measure K_z^2 for two or more combinations of $N(x)$ and $N(y)$ in each lattice to determine both parameters. As many as six and as few as two combinations of $N(x)$ and $N(y)$ were used for particular lattice pitches. The value of λ at a given lattice pitch was taken as the value for which the standard deviation of the mean value of B_m^2 (Equations (2) and (3)) was a minimum. This procedure assumes that both B_m^2 and λ are independent of core size or shape. The lattices for all of these measurements were either square or rectangular in cross section.

An alternative method of determining λ is to fit measured transverse flux shapes to the appropriate function, such as $\cos \frac{\pi r}{R}$. This method was not used in the present experiments because of the limited number of equivalent lattice points at which to place neutron detectors. Previous experiments with small-diameter rods of a different enrichment had shown that the two methods gave equivalent results[2].

It should be noted that the extrapolation distance determined experimentally is an 'effective' value, since the actual neutron boundary will conform roughly to the contour of the outer rods of the lattice. The values of the extrapolation distance determined for the 2- and 3-in. rods are presented in Figure 1.

The material buckling was calculated from the measured K_z^2 by (2), using appropriate values of the extrapolation distance taken from the smooth curves of Figure 1. Tables I and II list the results

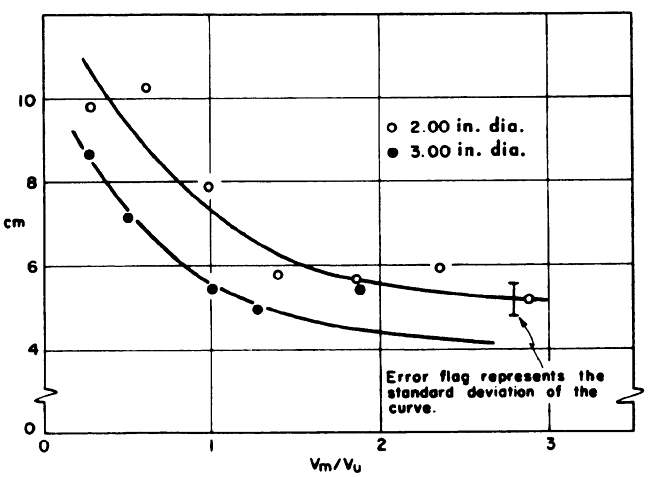

Fig. 1. Extrapolation distance versus V_m/V_u for 3.00 wt % U metal rods with 2- and 3- in. diameters in H_2O.

[2]H. KOUTS and R. SHER, "Experimental Studies of Slightly Enriched Uranium, Water Moderated Lattices. Part I. 0.600-In.-Diameter Rods," BNL-486 (T-111) (September 1957).

TABLE I

Summary of Exponential Measurements on 3 Wt % U^{235} Metal Rods, 3 in. in Diameter

Square Lattice Pitch, In.	V_m/V_u	Core Size	λ cm	B^2_{xy}, M^{-2}	K_z^2, M^{-2}	B_m^2, M^{-2}	Mc, lb of U
3.00	0.273	2 × 3	8.60	151.81	117.66	34.1	
		3 × 3		120.60	82.47	38.1	
		4 × 4		85.38	43.56	41.8	
		4 × 5		74.50	41.11	33.4	
		3 × 6		85.91	50.26	35.7	
					Avg.	36.6	11 100 ± 600
3.25	0.496	4 × 4	7.20	87.50	16.71	70.8	
		4 × 5		75.65	4.61	71.0	
					Avg.	70.9	3 170 ± 100
3.50	0.738	4 × 4	6.40	84.40	-3.47	87.9	1 990 ± 50
3.75	0.990	3 × 3	5.60	124.64	28.52	96.1	
		3 × 4		106.89	6.25	96.6	
					Avg.	96.4	1 610 ± 40
4.00	1.270	3 × 4	5.00	98.62	7.68	90.9	
		3 × 5		86.84	-4.15	91.0	
					Avg.	91.0	1 680 ± 50
4.50	1.860	4 × 4	4.40	66.34	7.20	59.1	
		4 × 5		55.84	-2.34	58.2	
					Avg.	58.7	2 980 ± 100
4.75/4.50	2.020	4 × 5[a]	4.40	53.33	6.67	46.7	4 180 ± 220
5.00/4.50	2.190	4 × 5[a]	4.30	51.70	17.05	34.7	6 420 ± 340
5.25/4.50	2.340	4 × 5[a]	4.20	50.22	26.25	24.0	11 200 ± 830

[a]Rectangular unit cell; pitch maintained at 4.50 in. along 5-element dimension, varied along 4-element dimension.

for the 3- and 2-in. rods, respectively. Figure 2 is a graphical presentation of the data for both rod sizes. The maximum material buckling, optimum volume ratio of moderator to metal (V_m/V_u) and infinitely safe volume ratio may be determined from this figure. The optimum volume ratio is that which yields the maximum buckling. The infinitely safe volume ratio is that for which an infinite number of rods would be critically safe. The latter is the ratio at which $B_m^2 = 0$.

Under the same assumptions that have been made previously, a critical mass may be calculated if the material buckling is known. The minimum critical mass is always associated with a spherical shape; therefore, the critical mass may be written as

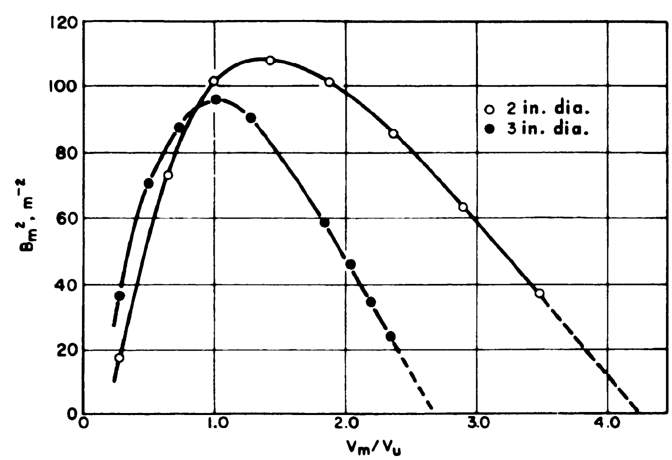

Fig. 2. Material buckling versus V_m/V_u for 3.00 wt % U metal rods with 2- and 3-in. diameters in H_2O.

TABLE II

Summary of Exponential Measurements on 3 Wt % U^{235} Metal Rods, 2 in. in Diameter

Square Lattice Pitch, in.	V_m/V_u	Core Size	λ cm	B_{xy}^2 M^{-2}	K_z^2, M^{-2}	B_m^2, M^{-2}	Mc, lb of U
2.00	0.273	4 × 4	10.70	113.28	97.64	15.6	
		4 × 5		101.65	88.22	13.4	
		4 × 6		93.27	71.15	22.1	
		4 × 7		87.03	68.73	18.3	
		4 × 8		82.25	63.18	19.1	
					Avg.	17.7	36 000 ±4 000
2.25	0.661	5 × 5	8.80	92.40	18.42	75.8	
		5 × 6		82.78	10.71	72.1	
		5 × 7		75.88	4.40	71.5	
					Avg.	73.1	2 280 ± 70
2.50	0.990	4 × 4	7.30	129.64	26.76	102.9	
		4 × 5		112.76	7.99	104.8	
		4 × 6		101.71	3.32	98.4	
					Avg.	102.0	1 180 ± 30
2.75	1.430	3 × 6	6.25	121.40	12.50	108.9	
		3 × 7		114.25	5.01	109.2	
		3 × 8		109.19	0.69	108.5	
		3 × 9		105.46	-2.64	108.1	
		4 × 4		120.60	11.11	109.5	
		4 × 5		104.14	-6.43	110.6	
					Avg.	109.1	980 ± 30
3.00	1.870	3 × 4	5.70	140.22	38.05	102.2	
		4 × 4		112.44	9.88	102.6	
					Avg.	102.4	990 ± 30
3.25	2.360	3 × 4	5.30	130.44	43.23	87.2	
		4 × 4		103.54	18.71	84.8	
					Avg.	86.0	1 210 ± 30
3.50	2.900	3 × 4	5.22	118.18	54.77	63.4	
		4 × 4		93.18	29.72	63.5	
					Avg.	63.4	1 790 ± 60
3.75	3.480	3 × 4	5.10	107.75	70.42	37.3	3 850 ± 190

$$Mc = \frac{4\pi\rho\left[\frac{\pi}{\sqrt{B_m^2}} - \lambda\right]^3}{3(1 + V_m/V_u)}$$

where ρ = density of U metal = 18.9 g/cm^3. Both λ and B_m^2 are functions of the moderator-to-metal volume ratio and their values may be obtained from the curves in Figures 1 and 2, assuming that λ has no core-shape dependence. The values determined for Mc in this way are shown graphically in Figure 3 and are also listed in Tables I and II.

To estimate the uncertainty in the critical mass obtained in the SRL experiments, it is sufficient to obtain the standard deviation of the mean value of the material buckling for each lattice. There are three cases in the Tables for which this buckling was determined for at least five different lattice shapes. These three cases are the 3-in. rods at 3.0-in. pitch and the 2-in. rods at pitches of 2.0 and 2.75 in. The standard deviation of the mean value in these three cases is, respectively, 1.50, 1.49 and 0.35 m^{-2}. The average value of these three numbers, 1.1 m^{-2}, was used as the standard deviation for all the material bucklings in calcu-

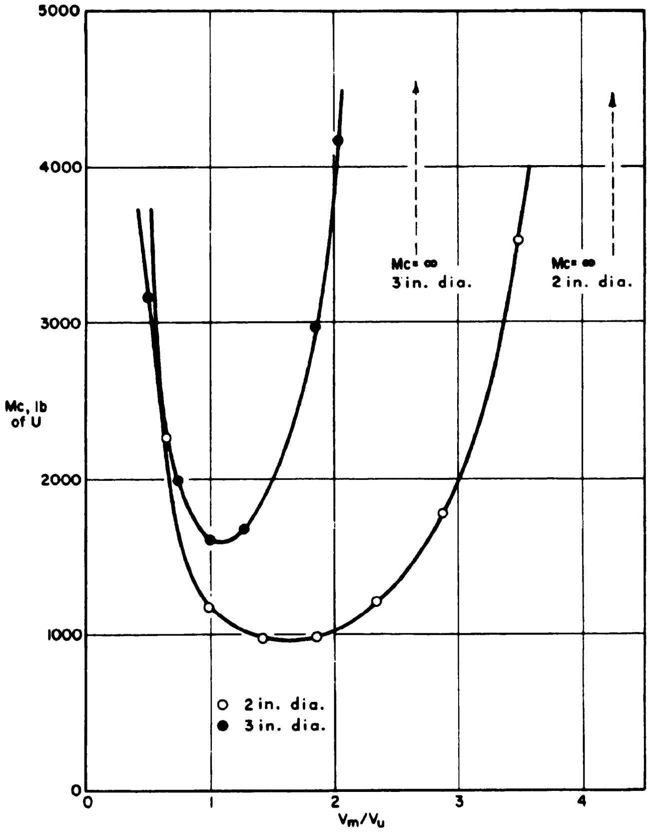

Fig. 3. Critical spherical mass versus V_m/V_u for 3.00 wt % U metal rods with 2- and 3- in. diameters in H_2O.

lating the standard deviation of the critical masses quoted in the Tables.

An alternative approach is to consider the individual uncertainties in K_z and in λ separately. Each value of K_z given in the Tables is the average of at least four determinations for that lattice,

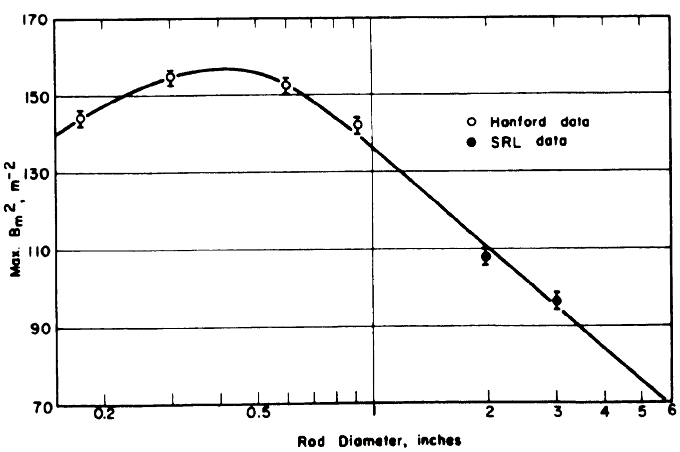

Fig. 4. Maximum buckling versus rod diameter 3.0 wt % U metal rods in H_2O.

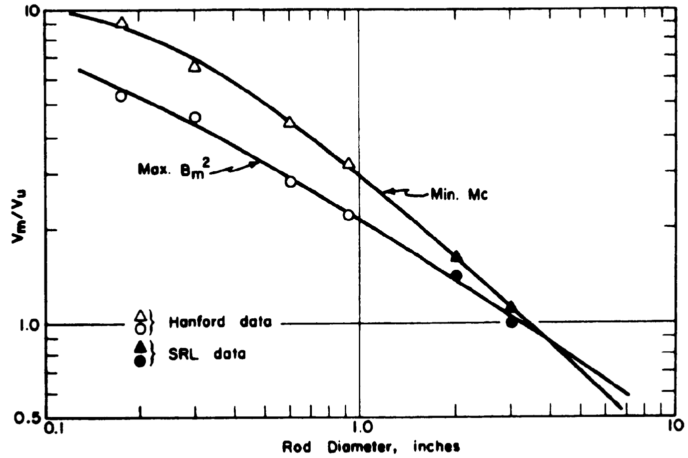

Fig. 5. Optimum value of V_m/V_u to yield maximum buckling or minimum critical mass for 3.0 wt % U metal rods in H_2O.

with a standard deviation of the mean not exceeding $\pm$ 0.2 m^{-2}. Uncertainties in the extrapolation lengths used therefore constitute the bulk of the uncertainties in the bucklings. The values of the extrapolation lengths used were obtained by minimizing the standard deviations in the bucklings at a given lattice pitch, and fitting the results to a smooth curve. Inspection of this curve in Figure 1 indicates that the standard deviation of any point does not exceed $\pm$ 0.5 cm. The possibility of systematic errors in these values cannot, however, be completely excluded.

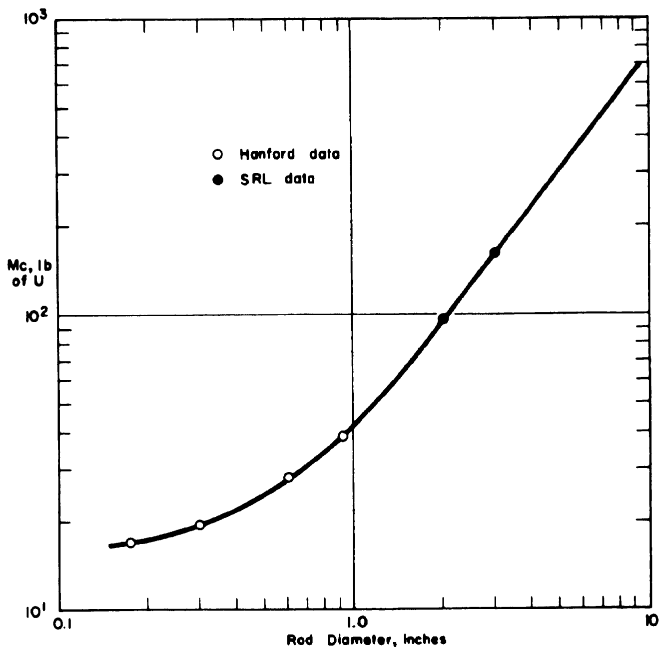

Fig. 6. Minimum critical spherical mass versus rod diameter 3.0 wt % U metal rods in H_2O.

COMPILATION OF ALL 3 WT % U^{235} METAL ROD DATA

Table III summarizes those parameters needed for establishing standards for criticality safety in handling, storing and shipping 3 wt % U^{235} metal for the rod sizes measured at Hanford and SRL. Figures 4 to 6 plot maximum material buckling, optimum V_m/V_u ratios for minimum critical mass and material buckling, and minimum critical mass as a function of rod diameter. No attempt was made to correct for small differences in U^{235} enrichment of the rods. (Hanford rods were 3.06 wt % U^{235}; SRL rods were 3.00 wt % U^{235}.) Straight-line extrapolations can be used in Figures 4 to 6 with a good degree of confidence to obtain critical data on rods of larger diameter, such as 7-in.-diameter casting billets.

TABLE III

Summary of Parameters for 3 Wt % U^{235} Metal Rods

Rod Diameter, in.	Optimum V_m/V_u to yield—		Max. B_m^2, M^{-2}	Min. Mc, lb of U
	—Max. B_m^2	—Min. Mc		
0.175[a]	5.3	9.0	144.0	170
0.300[a]	4.5	6.9	154.5	194
0.600[a]	2.8	4.3	152.5	282
0.925[b]	2.2	3.2	142.2	387
2.000	1.4	1.5	109.1	960
3.000	1.0	1.1	96.7	1600

[a]Reference (1), approach-to-critical and exponential methods.
[b]Reference (1), exponential method.