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Exponential Experiments with Slightly Enriched Uranium Rods in Ordinary Water

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EARLIER WORK

Light water was one of the first moderators considered for nuclear reactors. Early calculations, however, led to the belief that the relatively large thermal neutron absorption by hydrogen would not permit its use in obtaining a chain-reacting system with natural uranium.

By 1944, the experimental values of cross sections had changed enough to cause the use of water to be reconsidered. Calculations done by A. M. Weinberg[1] still indicated that criticality could not be reached. However, it seemed much closer than was thought true before, and uncertainties in the analysis were still large enough to allow the possibility that k_∞ would be greater than 1. An experimental program to settle the question was therefore carried out in 1944–45, at Oak Ridge.

The Oak Ridge measurements were made in exponential experiments, with natural uranium rods of diameter 1.18 in. (3.00 cm), 1.10 in. (2.794 cm), and 0.787 in. (2.00 cm). Several volume ratios of water to uranium were assembled with each rod size. Although the experiments were primarily meant to provide values of the buckling B^2, there were also measurements of migration areas and temperature coefficients of reactivity. In addition, an attempt was made to find values of the thermal utilizations of some of the lattices.

The exponential experiments were assembled in a tank which rested on a thermal column on top of the Oak Ridge graphite reactor. In each case the basic cell was an equilateral triangle, and the lattice was rectangular in cross-section. The water tank had inlet and outlet vents, for filling and emptying; these also permitted a continuous flow in and out of hot water for temperature coefficient measurements.

Calculations of the thermal utilization had led to the prediction that an annular air gap about the fuel rods would increase the maximum attainable k. Mathematically this effect is a consequence of the boundary conditions which the flux must satisfy at the fuel-moderator interface. Physically, it results from the fact that scattering material has been removed from the volume near the boundary of the fuel rods, where it would have deflected neutrons away from the uranium. Three of the exponential assemblies were made with rods having air gaps of this sort, so that the reality of the effect and its magnitude could be observed. The air gaps were obtained by sealing each uranium rod in two concentric aluminum tubes, separated by aluminum spacing elements.

The buckling of each lattice was found from indium foil measurements of the spatial thermal neutron distribution. These foils were wrapped around plastic rods, which were inserted into the assembly in the centers of lattice cells. Flux traverse measurements were made along the three principle axes of the rectangular assembly, and the observed variations in foil activities were fitted to the appropriate cosine and exponential functions. All measurements were limited to the region higher than 50 cm above the surface of the thermal column, in order that flux harmonics might be avoided. For most of the assemblies, buckling measurements were made with two or more water temperatures. Thus values of dB^2/dt were obtained, as well as B^2 at room temperature.

Table I lists the experimental values of B^2 at 25°C and the observed temperature coefficients. Figure 1

Table I. Buckling and Temperature Coefficients of Light Water Moderated, Natural Uranium Rod Lattices. From 1944–45 Experiments at Oak Ridge

Rod diameter (in.)	Air gap (in.)	$\frac{\text{Volume water}}{\text{Volume uranium}}$	Buckling B^2 ($cm^{-2} \times 10^4$) at 25°C	$\frac{dB^2}{dT}$ ($cm^{-2}/°C \times 10^5$)
1.18	None	1.14	− 5.34	
1.18	None	3.27	−42.15	+1.50
1.18	None	0.273	−67.92	−2.3
1.18	None	1.78	− 3.79	+0.20
1.18	None	1.36	− 1.98	+0.002
1.18	None	3.17	−34.95	+0.817
1.10	None	2.06	− 8.01	+0.31
1.10	None	1.57	− 3.16	+0.159
1.10	None	1.42	− 3.39	−0.034
1.10	0.45	1.92	− 0.20	−0.089
1.10	0.45	1.56	+ 0.27	
0.787	None	1.80	− 2.78	
0.787	None	1.50	− 6.60	−0.25
0.787	None	2.10	− 3.29	
0.787	0.354	2.01	0.00	+0.016

* Brookhaven National Laboratory. Prepared by H. Kouts. Including work by T. Arnette, S. Bernstein, G. Branch, F. Beyerly, E. Cashwell, C. Clifford, I. Coveyou, B. Feld, Haydn Jones, R. McCord, H. W. Newson, W. Neyer, F. V. Pruitt, R. B. Stewart, J. R. Rush, R. Scalletar, L. Slotin, L. B. Watson, A. M. Weinberg—Clinton Laboratories (Now Oak Ridge National Laboratory).

shows the buckling plotted against the water-to-uranium volume ratio for the 1.10-in. and 1.18-in. rods. These sizes would have nearly the same reactivity, and so they have been grouped together on the graph. Figure 2 shows the experimental values of B^2 for the 0.787-in. rods again plotted against the volume ratio.

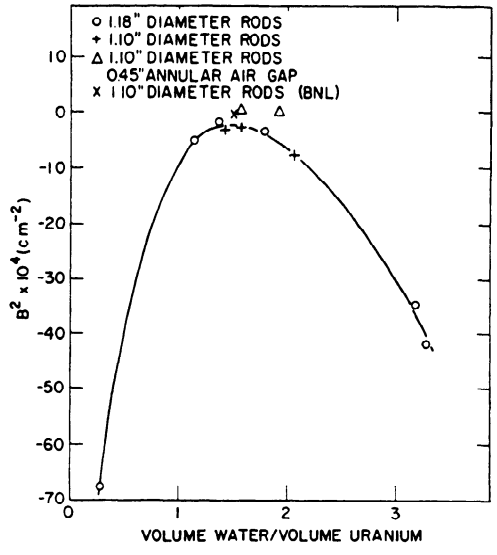

Figure 1. B^2 vs volume ratio, 1.10 inch and 1.18 inch diameter natural uranium rods in light water. From 1944–45 Oak Ridge experiments

According to Fig. 1, the maximum value of B^2 obtainable with the larger rods was about -3×10^{-4}, at a water-to-uranium volume ratio of about 1.5. From Fig. 2 it is seen that the 0.787-in. diameter rods gave also a maximum value of about -3×10^{-4} for B^2, but the peak value in this case occurred at a volume ratio of about 1.9.

The annular air gaps did indeed increase the reactivity; in fact one lattice (1.10-in. diameter rods, 0.45-in. air gap on the radius, 1.56 volume ratio) had a positive value of the buckling after an allowance had been made for absorption in the aluminum tubing and spacers.

For measurements of the age of fission neutrons in these assemblies, the uranium rods were wrapped in cadmium. A source of fission neutrons was applied, and cadmium-covered indium foils were exposed throughout the lattice. The observed activities of these foils as a function of the distance r from the source led to the evaluation of the mean neutron age from fission energies to the large indium resonance at 1.46 electron volts. The source used to find the distribution for small values of r was a small sample of enriched uranium, effectively a point. A beam of neutrons through a hole in the reactor shield produced the necessary fissions. The source used for large values of r was a plane array of Oak Ridge pile slugs, each 1.10 in. in diameter by 4.0 in. long, placed under the lattice in the tank on the thermal column. The space distribution observed with the plane source was differentiated to convert it to a point source curve, and the result was joined onto the measured point source distribution to give a single extended curve $f(r)$. The age to the indium resonance τ_{In} was then calculated from

$$\tau_{In} = \frac{1}{6}\overline{r^2} = \frac{1}{6}\frac{\int_0^\infty r^4 f(r)\,dr}{\int_0^\infty r^2 f(r)\,dr} \qquad (1)$$

A calculated correction had to be added to convert τ_{In} to τ_{th} the age to thermal. The values of τ_{th} which were obtained (Table II) are plotted in Figs. 3 and 3a against the water-to-uranium volume ratio.

The purpose of the cadmium wrapped around the fuel rods was to suppress fissions in the lattices. We now believe that such a precaution was not sufficient; the fast fission effect and the fraction of fissions in U^{235} produced above the 0.4 electron volt cadmium cutoff probably caused the measurements to be wrong. The effect of epi-cadmium fissions would have been to spread out the source supplying the neutrons measured at the indium resonance, and this in turn would have led to values of τ_{th} which were too high.

As mentioned before, the primary purpose of the exponential experiments was to show if it were possible to make a chain reacting system with natural uranium rods and light water. It appeared from the results that it might indeed be possible to do so, if suitable annular air gaps were placed about the fuel rods. However, it seemed unlikely that even tricks of this sort could provide enough excess reactivity to overcome the xenon poisoning which would exist in a high flux reactor.

It was pointed out in the final report on the Oak Ridge experiments that a very small increase in enrichment of the U^{235} content of the uranium used would make a reactor moderated with light water feasible. It was this point which later prompted the Brookhaven program of exponential experiments with light water and slightly enriched uranium rods.

BROOKHAVEN MEASUREMENTS WITH 0.750-IN. DIAMETER, 1.027% ENRICHED URANIUM RODS

The first series of Brookhaven exponential experiments was done with approximately 1.8 tons of uranium, enriched to 1.027 $\pm$0.001% U^{235} by weight

Table II. Neutron Age from Fission to Thermal Energies. From 1944–45 Oak Ridge Experiments

Rod diameter (in.)	*Air gap (in.)*	*Volume water / Volume uranium*	*τ_{th} (cm^2) at 25°C*
1.18	None	1.14	54.7
1.18	None	3.27	39.9
1.18	None	0.273	131.0
1.18	None	1.78	44.0
1.18	None	1.36	51.7
1.18	None	3.17	40.1
1.10	None	2.06	44.9
1.10	None	1.57	49.2
1.10	None	1.42	51.0
1.10	0.45	1.92	94.5
1.10	0.45	1.56	112.0
0.787	None	1.80	48.4
0.787	None	1.50	51.5
0.787	None	2.10	46.5
0.787	0.354	2.01	110.0

(95.16 atoms U^{238} per atom of U^{235}). The metal was fabricated into 0.750 $\pm$0.001-in. diameter cylindrical rods, four feet long. These were straightened to within 0.015-in. lateral deviation in the four-foot length; they were then clad with 0.030 in. of aluminum, drawn on to ensure a tight fit. Each end of the tubing was capped with an aluminum plug, to provide a water-tight containment of the uranium.

The experiments were carried out in a water tank placed over a thermal column which occupies a portion of the top of the Brookhaven pile shield. This thermal column is made of five one-foot-thick layers of graphite, each slightly wider than the one below. The stepped design prevents fast neutron leakage up the sides of the thermal column, and the well-machined surfaces of the graphite blocks which make up the structure give a void-free construction which ensures a well-moderated source of neutrons at the top surface. The cadmium ratio of this source as measured with 0.005-in. thick indium foils is better than 10^5.

The water tank is cylindrical, six feet in diameter and six feet high. Surrounding it is a crude radiation shield made of steel plates and heavy concrete.

Figure 4 shows the structure which was used to support and position the uranium rods. The top of this structure was a steel ring, inside the hole in the center of which was inserted a 1.5-in. thick aluminum disc, with holes drilled in it to space the uranium rods for the water-to-uranium volume ratio desired. A similar disc was put at the bottom of the support structure, so that the spacing between rods was maintained at both ends. The top and bottom spacing elements were separated by an aluminum cylinder, perforated so that water could get in and out freely. Since the fuel rods were suspended from the top fuel rod locating plate, the whole structure could be removed from the water tank as a unit. This feature permitted assembling the experiments remotely, and then transferring the loaded lattice by crane to the water tank.

Distilled water was used as the moderator. There were occasional spectroscopic tests of the purity, and there were frequent tests of reproducibility of the data. During a large part of the experiments the water contained large amounts of boric acid (for migration area measurements). Sheets of cadmium were inserted to suppress fissions when measurements were not being done. Because of the possibility that boron or cadmium might plate onto the fuel rods, reproducibility was sought and maintained throughout the experiment.

The radius of a loaded lattice was at most about 40 centimeters, whereas the radius of the water tank was about 90 centimeters. Thus the experiments were done with an effectively infinite water reflector. This had the effect of making the 1.8 tons of uranium equivalent to about 1.4 times the mass in an unreflected experiment.

Altogether six volume ratios of water-to-uranium were assembled. There were measurements of the buckling B^2, the migration area M^2, the fast effect and the thermal utilization f.

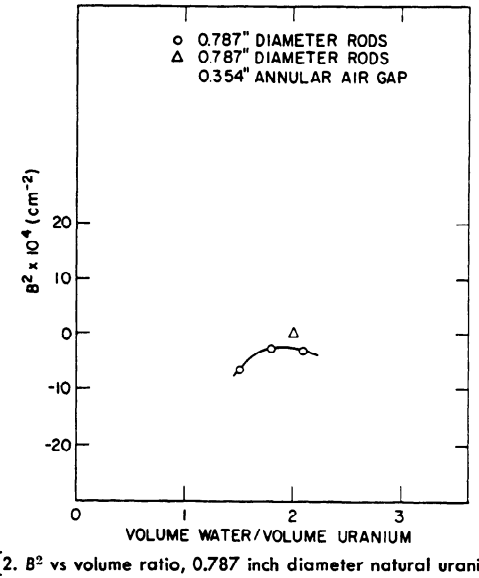

Figure 2. B^2 vs volume ratio, 0.787 inch diameter natural uranium rods in light water. From 1944–45 Oak Ridge experiments

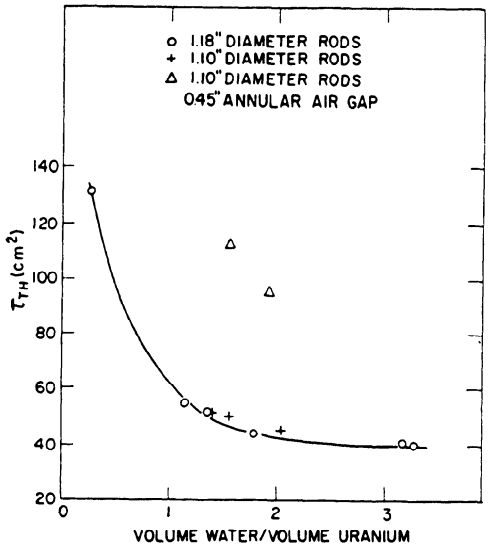

Figure 3. τ_{th} vs volume ratio, 1.10 inch and 1.18 inch diameter natural uranium rods in light water. From 1944–45 Oak Ridge experiments

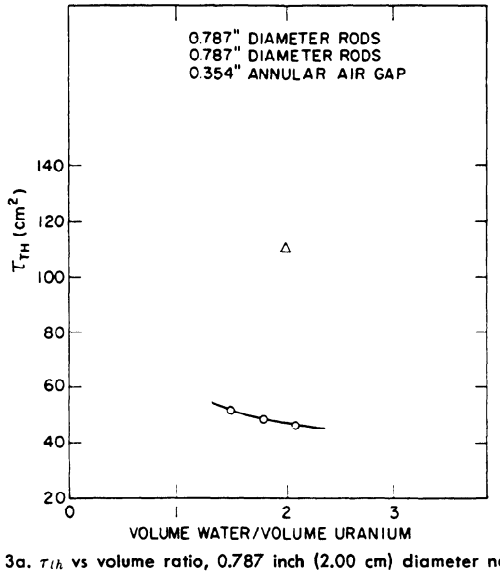

Figure 3a. τ_{th} vs volume ratio, 0.787 inch (2.00 cm) diameter natural uranium rods in light water. From 1944–45 Oak Ridge experiments

Buckling

In an exponential experiment the buckling is usually found from differentiation of the measured thermal neutron flux distribution. For a cylindrical array such as was used here, the space dependence of the flux can be written as a series of Bessel functions:

$$\varphi = \sum_{n=1}^{\infty} J_0\left(a_n \frac{r}{R+\lambda}\right) f_n(z) \qquad (2)$$

where r and z are cylindrical coordinates in the lattice, λ is the reflector savings, R is the loaded radius, and a_n are the successive roots of

$$J_0(r) = 0 \qquad (3)$$

The coefficients $f_n(z)$ have the form

$$f_n = C_n \exp\left[\left\{\left(\frac{a_n}{R+\lambda}\right)^2 - B^2\right\}^{\frac{1}{2}} s\right] + D_n \exp\left[-\left\{\left(\frac{a_n}{R+\lambda}\right)^2 - B^2\right\}^{\frac{1}{2}} s\right] \qquad (4)$$

C_n is small compared with D_n, since the first term is only important near the upper boundary of the lattice. Furthermore, the rates of decay of the harmonics ($n > 1$) are greater than those of the fundamental, and so above some value $s = z'$ the flux can be represented accurately by the fundamental alone:

$$\varphi \sim A_1 J_0\left(a_1 \frac{r}{R+\lambda}\right) f_1(z), \; z > z' \qquad (5)$$

Ordinarily, one would measure the radial and axial flux distributions in the region of validity of Equation 5. The radial distribution would be fitted to $J_0(a_1 r/r_0)$, the axial distribution would be fitted to $C_1 \exp[z/L] + D_1 \exp[-z/L]$, and the buckling would be obtained as

$$B^2 = \left(\frac{a_1}{r_0}\right)^2 - \frac{1}{L^2} \qquad (6)$$

However, the number of fuel rods used in these experiments did not permit accurate measurements of the radial flux distribution. Symmetry conditions made only five positions available for flux measurements along any one radius, and the outermost of these was so near the water reflector as to make it useless. Thus only four points were available along any one radius, and these were not enough to provide any real accuracy.

On the other hand, the axial flux distributions showed in every case the behavior of the presence of the fundamental only, above about 10 cm above the bottom of the lattice. Moreover, corrections for the effect of the top boundary (the first term of Equation 4) were small below about 70 cm from the top of the lattice. Thus there was a region of about 60 cm in z over which the flux was well represented by an exponential, and so good values of L could be determined. It was therefore decided to base evaluation of B^2 on measurements of L alone.

To determine B^2 this way, one would measure L for a wide range of rod loadings. Each such loading would be idealized as a cylinder having a radius R defined by

$$R = b\sqrt{N} \qquad (7)$$

N being the number of rods loaded, and πb^2 being the area of a single lattice cell. Least squares fits would then be made to λ and B^2 in the expression

$$L = \left\{\left(\frac{a_1}{R+\lambda}\right)^2 - B^2\right\}^{-\frac{1}{2}} \qquad (8)$$

Such an analysis contains an implicit assumption that λ and B^2 have the same value for all pairs of values of L and R. This method of finding B^2 had never been used before, and so there remained some doubt as to the validity of the assumption just mentioned. A conclusive test could have been provided by the performance of a few critical assemblies, but these could not be done because the amount of uranium fabricated was not sufficient. Thus the accuracy of the method was not confirmed until later, when measurements were performed with a slightly higher enrichment of fuel.

The measurements of neutron flux distributions were made with indium foils, 0.220 in. in diameter and 0.005 in. thick. These were placed in foil-holding rods inserted through the top fuel-locating plate. At first the foil rods used were thin-walled aluminum tubes, with slots punched out at 10 cm intervals. The foils were placed between aluminum covers, and then wedged in the slots. Later the aluminum tubes were replaced with solid plastic (methyl methacrylate) rods, having in them machined depressions for holding the indium foils. This kind of foil rod could be constructed much more accurately than the former, and except for a high replacement rate because of breakage, it proved much more satisfactory.

Because of the large local variation of flux, the foils had to be centered accurately in lattice cells. This was accomplished by the use of plastic spacers located at 30 cm intervals throughout the height of the assembly.

A careful analysis was made of foil-counting in-

Figure 4. Brookhaven exponential experiment assembly

accuracies; these included both random and systematic errors from foil counting methods, the statistics of foil counting, the weight variations of the foils, and the uncertainties in separating foils during exposure to the neutron flux. According to this analysis, the predicted uncertainties in individual foil counts were at most about 1.5 per cent. The observed standard deviations showed slightly less error than this.

All values of the relaxation lengths were obtained from least-squares fits of exponentials to the observed axial flux variations.

Values of B^2 were found for water-to-uranium volume ratios 1.334, 1.584, 1.834, 2.334, 2.834, and 3.834. In each case the volume ratio given was that of the actual volume of water to that of uranium, the aluminum being ignored. The experimentally determined values of B^2 and λ are given in Table III, and are plotted in Figs. 5 and 6. The experimental errors included are derived from the standard deviations of the least squares fits which were made; thus they may not include some systematic errors.

Table III. Experimental Values of Buckling and Reflector Savings, 0.750-in. Diameter Rods of 1.027% Enriched Uranium in Light Water

$\frac{\text{Volume water}}{\text{Volume uranium}}$	B^2 ($cm^{-2} \times 10^4$)	λ (cm)
1.334	28.9 ± 0.5	7.71 ± 0.14
1.584	34.7 ± 0.3	7.16 ± 0.10
1.834	37.5 ± 0.8	6.94 ± 0.23
2.334	36.7 ± 0.5	6.90 ± 0.16
2.834	32.9 ± 0.2	6.94 ± 0.11
3.834	18.6 ± 0.6	6.42 ± 0.22

Figure 5 has the characteristic shape of a buckling curve for a thermal reactor. Heating these lattices would expel water, and hence decrease the water-to-uranium volume ratio. Therefore one would expect a zero temperature coefficient near the peak of the buckling curve, with a negative coefficient at lower volume ratios and a positive coefficient for higher volume ratios. For these lattices the peak is at a volume ratio of about 2, and the safe design region is below this point.

If the slowing-down and diffusion of neutrons does not take place isotropically in a reactor core, the interpretation of associated exponential experiments must be modified accordingly. The effect has been considered by G. Young and J. Wheeler; they showed that if the anisotropy is large, it is possible to draw grossly inaccurate conclusions from a simple exponential experiment.

As will be seen later, the experimental values of the migration area are nearly constant over the range of volume ratios studied, and are not very different from the value of M^2 for fission neutrons in pure water. Thus it appears that because of large inelastic scattering, uranium in water can be considered to be very nearly equivalent for neutron slowing-down to the water displaced. This feature would make the existence of anisotropy very unlikely. Nevertheless, it appeared worth-while to determine the effects of whatever non-isotropic processes do occur.

The anisotropy measurement which was performed was originally suggested by E. P. Wigner. Its principle can be illustrated by considering a subcritical lattice in the shape of a parallelepiped, with edges $l_x = l_y; l_z$. The fuel elements are assumed to be parallel to the z axis. If the thermal neutron source activating this assembly is placed at one end of the lattice (with edges l_x and l_y), the one-group critical equation is

$$k_\infty - 1 = M_\perp^2(B_x^2 + B_y^2) - M_\parallel^2 B_z^2 \qquad (9)$$

$M_\perp^2$ and $M_\parallel^2$ have physical meaning associated with the mean square distance a fission neutron travels in directions perpendicular to and parallel to the rods, respectively. B_x^2 and B_y^2 are defined by

$$B_x^2 = B_y^2 = \frac{\pi^2}{(l_x/2 + \lambda)^2} \qquad (10)$$

and

$$B_z^2 = 1/L^2 \qquad (11)$$

where L is the relaxation-length for decay of the fundamental in the z direction.

If, on the other hand, the source is placed on one of the sides of the lattice (that defined by y and z), in place of Equation 9 one would have

$$k_\infty - 1 = M_\parallel^2 b_z^2 + M_\perp^2(b_y^2 - b_x^2) \qquad (12)$$

where

$$b_z^2 = \frac{\pi^2}{(l_z/2 + \lambda)^2} \qquad (13)$$

$$b_y^2 = B_y^2 \qquad (14)$$

$$b_z^2 = 1/(L')^2 \qquad (15)$$

L' is now the relaxation length of the fundamental in the x-direction. From Equations 9 and 12,

$$M_\perp^2 M_\parallel^2 = \frac{B_z^2 + b_z^2}{B_x^2 + b_x^2} \qquad (16)$$

A "double lattice" measurement of this sort was carried out with the 1.334 volume ratio lattice, with the result

$$M_\perp^2/M_\parallel^2 = 1.039 \pm 0.028 \qquad (17)$$

Thus the anisotropy found is just outside the error limits of the experiment. It was therefore neglected.

Migration Area

The measurements of M^2 were based on its definition by the critical equation; for reasons to be discussed in connection with later measurements, this was taken to have the form

$$k_\infty = \exp[M^2 B^2] \qquad (18)$$

Since the assemblies studied were heterogeneous, k_∞ could also be expressed through the four-factor formula

$$k_\infty = \epsilon p f \eta \qquad (19)$$

Thus from equating Equations 18 and 19,

$$\epsilon p f \eta = \exp[M^2 B^2] \qquad (20)$$

Table IV. Experimental Values of Migration Areas for 1.027% Enriched, 0.750-in. Diameter Uranium Rods in Light Water

$\frac{Volume\ water}{Volume\ uranium}$	M^2 (cm^2)
1.334	28.9 ± 1.7
1.834	28.2 ± 1.2
2.834	27.4 ± 0.4
3.834	27.5 ± 1.1

The measurement consisted in dissolving boric acid in the moderator water, and measuring the effect of this neutron poison on the buckling. Aside from B^2, the only other quantity in Equation 20 which would be affected appreciably by the additional neutron absorber is f; M^2 would also be decreased to some extent through the change in diffusion length, but it can be shown that within the error of the measurements, this effect can be neglected.

Thus as successively increasing amounts of boron were added to the moderator water, the value of f and B^2 changed together so as to maintain the correctness of Equation 20. At each boron concentration, B^2 was measured by the method discussed above. Also, the boron concentrations were measured, and the thermal utilizations were calculated.† M^2 could then be determined as the slope of the straight-line plot of B^2 against $\ln f$.

The best straight-line fits of B^2 vs $\ln f$ were obtained by the method of least squares. The values of the migration area so obtained are given in Table IV, and are plotted in Fig. 7. The measurement was carried out for four of the six volume ratios studied.

A comparison of Fig. 7 with Figs. 3 and 4, results of Oak Ridge measurements of τ_{th}, shows striking differences. The Brookhaven values are smaller at all volume ratios; furthermore, they do not show the rapid increase at smaller volume ratios which was seen in the Oak Ridge measurements. Further Brookhaven experiments with smaller rods (discussed later in this paper) tend to substantiate the low values of M^2, and they are believed to be correct.

Several causes could have contributed to the high values seen in the course of the Oak Ridge experiments. One of these, mentioned earlier is that the measurements depended on complete suppression of fissions by the cadmium sheathing of the rods. Actually, the cadmium could only have eliminated fissions caused by neutrons with energies below about 0.4 electron volts, and could not have noticeably influenced neutrons with higher energies. Experiments not discussed here indicate that the fraction of fissions caused by neutrons with energies above the cadmium cut-off rises from about 0.11 at a volume ratio of 4 to about 0.38 at a volume ratio of 1. Thus the cadmium used at Oak Ridge could not have performed its function. The effect of epithermally induced fissions would have tended to smear out the neutron source, thus invalidating the experiment. As the volume ratio decreased, the per cent of disturbing epithermal fissions increased; therefore the Oak Ridge values of migration areas must be considered to be least reliable at small volume ratios, where they do indeed differ most from the Brookhaven values.

Another point which must be considered is that the quantity sought in the Oak Ridge measurements was defined by Equation 1; the quantity derived from Brookhaven measurements was defined by Equation 20. According to elementary homogeneous reactor theory, these two definitions should be equivalent, but there seems to be no good reason why they may not be somewhat different for a heterogeneous pile.

The difference between Brookhaven and Oak Ridge results is discussed also by J. Chernick, in another paper in these proceedings.[2] There both sets of measurements are considered in the light of recent calcula-

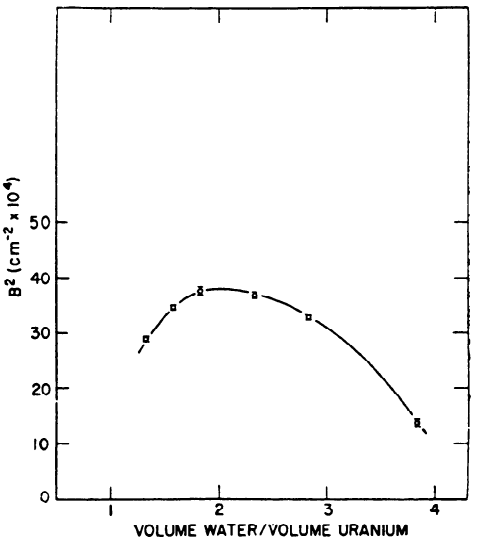

Figure 5. Experimental values of buckling for 0.750 inch diameter, 1.027% enriched uranium rods in light water

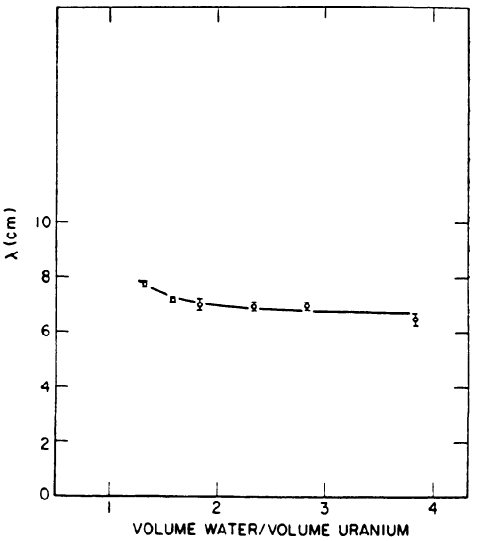

Figure 6. Experimental values of reflector savings for 0.750 inch diameter, 1.027% enriched uranium in light water

† Calculations by K. Puechl, Walter Kidde Nuclear Laboratory, using diffusion theory plus a correction for the P_3 content of the flux.

tions of the slowing down of neutrons in uranium-water lattices, and the conclusion is reached that the Brookhaven results compare reasonably well with computations using a reasonable value for the inelastic scattering cross section of uranium. The Brookhaven results also seem reasonable in the light of the most recent calculations of neutron slowing-down and diffusion in pure water.[3]

Fast Effect

Values of ϵ were measured for two of the lattices: those with the volume ratios 1.334 and 3.834. The method resembles one used by D. L. Hill at Los Alamos, in 1944.

The measurement consisted in finding the ratio of U^{238} fissions to U^{235} fissions in a typical fuel rod. The following simple analysis will show how this quantity, which we shall call F, is related to the fast effect.

ϵ is defined to be the number of neutrons slowing down past the U^{238} fission threshold, per neutron from thermal fission. Since it can be shown that the U^{235} fissions in the fast fission region are negligible, an equivalent definition is: ϵ is the number of neutrons slowing down past the U^{238} fission threshold, per neutron from U^{235} fission. Thus

$$\epsilon = (1 - n - r) + n\nu_{28}(1 - n - r) + m^2\nu_{28}^2(1 - n - r) + \cdots = \frac{(1 - n - r)}{1 - n\nu_{28}} \tag{21}$$

where n is the probability that an average fission neutron will cause fission in U^{238}, r is the probability that a fission neutron will be lost to other processes (radiative capture, fast neutron leakage, etc.) before slowing down past the U^{238} fission threshold, and ν_{28} is the average number of neutrons per U^{238} fission.‡

On the other hand, the number U^{238} fissions per neutron from thermal fission is

$$n + n^2\nu_{28} + n^3\nu_{28}^2 = \frac{n}{1 - n\nu_{28}} \tag{22}$$

and the number of thermal fissions on the same basis is of course

$$1/\nu_{25} \tag{23}$$

where ν_{25} is the number of neutrons per U^{235} fission. Thus the number of fast fissions per U^{235} fission is the ratio of Equation 22 to Equation 23; this is

$$F = \frac{n\nu_{25}}{1 - n_{28}} \tag{24}$$

(F is the quantity to be measured; it was mentioned above.)

If fast neutron leakage is neglected, one can write

$$r = \alpha n \tag{25}$$

where α is the neutron capture-to-fission ratio, averaged over the fission neutron spectrum and the U^{238} fission cross section. Then from (21), (24), and (26),

$$\epsilon - 1 = F\frac{\nu_{28} - 1 - \alpha}{\nu_{25}} \tag{26}$$

At the present time, the value of ν_{28} is somewhat uncertain. Old Los Alamos measurements by J. M. Blair and A. O. Hanson give

$$\frac{\nu_{28}}{\nu_{25}} = 0.983 \pm 0.55 \tag{27}$$

Recently there have been developments placing this result in doubt, and the experiment is being redone by J. L. Fowler at Oak Ridge and J. Terrell at Los Alamos. For the purpose of this analysis it has been assumed that

$$\nu_{28} = \nu_{25} = 2.5 \tag{28}$$

If, however, ν_{28} is noticeably different from this value, the deduced values of ϵ will be changed appreciably.

Since fast fissions in U^{238} are produced by neutrons which have nearly the energy with which they in turn were produced,

$$\alpha = \frac{\int_{E_0}^{\infty} dEN(E)\sigma_\gamma^{28}(E)}{\int_{E_0}^{\infty} dEN(E)\sigma_f^{28}(E)} \tag{29}$$

with $N(E)$ the fission neutron spectrum, σ^{28} the radiative capture cross-section, and E_0 the U^{238} fission threshold. All radiative capture below the U^{238} threshold is thus considered as part of the resonance capture.

Numerical integration of Equation 29 leads to

$$\alpha = 0.107 \tag{30}$$

Then from Equations 26, 28, and 30

$$\epsilon - 1 = 0.557F \tag{31}$$

The evaluation of ϵ rests on Equation 31, which still contains an uncertain value of ν_{28}.

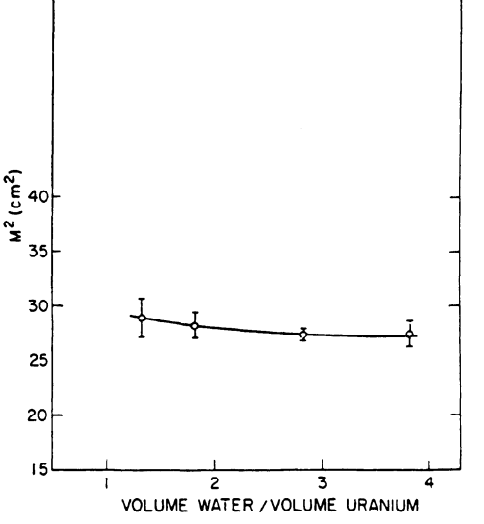

Figure 7. Experimental migration areas of lattices of 0.750 inch diameter, 1.027% enriched uranium in light water

‡ The abbreviation 28 dates back to the Manhattan Project. At that time it was customary to represent a nucleus of a fissioning species by a two-digit number, the first of which was the last digit of the atomic number and the last of which was the last digit of the atomic weight. Thus 28 is the element with atomic number 92 and atomic weight 238.

In these experiments, F was determined in the following way. A fuel rod was cut cross-wise into sections, and between these were placed sandwiches of aluminum and uranium foils. The aluminum foils served to capture fission products from the uranium, and the activities of these fission products led to a measure of the fission density in the uranium.

Two such sandwiches were used per measurement of ϵ. One consisted of three layers of 0.0007-in. thick aluminum on each side of a depleted uranium foil, this entire sandwich being compressed between two sections of the fuel rod. The depleted uranium foil contained only 7×10^{-6} parts of U^{235} per part of U^{238}. In the neutron flux of the exponential experiments, the U^{235} fission rate in the foil was less than 1% of the U^{238} fission rate; accordingly any fissions occurring in it could be attributed to U^{238} alone. The second foil sandwich was precisely the same as the first, except that the depleted uranium was replaced by an aluminum foil, 0.005 in. thick. The two sandwiches were separated by a section of uranium rod two in. long. The experimental arrangement of fuel rod and aluminum and uranium foils is shown in Fig. 8.

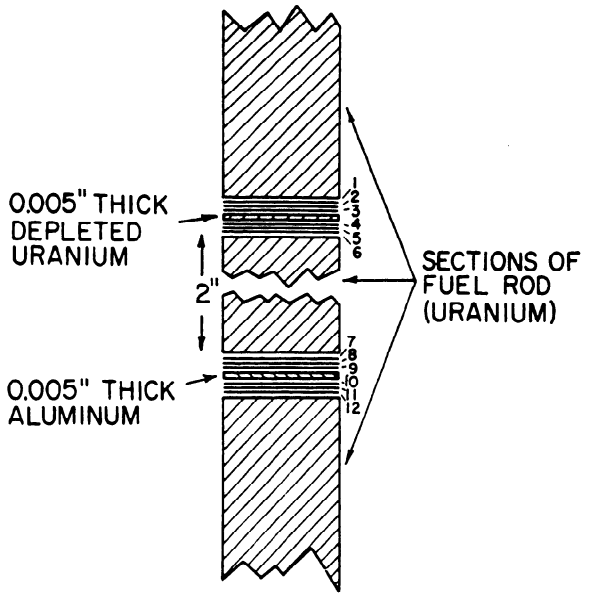

Figure 8. Schematic arrangement of fuel rod and uranium and aluminum foil used to measure fast effect

It was assumed that the fission densities in the section of fuel rod used were the same as if the uranium had not been cut and the foil sandwiches inserted. Actually, the foil sandwiches constituted a low absorption region through which thermal neutrons could stream; this effect increased the thermal fission rates locally. A simple calculation showed that this streaming could have altered the measured values of ϵ by at most about 0.01%; thus the effect was not observable.

The fuel rod used was inserted in a water-tight aluminum tube with 0.755-in. inner diameter and 0.785-in. outer diameter, and was put in the center of an exponential lattice, replacing a standard uranium rod. The entire lattice was then exposed to the neutron source for a period of about eighteen hours. At the end of this time the rod was removed, and the activities of the foils designated in Fig. 8 as 1, 3, 4, 6, 7, 9, 10, and 12 were determined with end-window β-ray counters.

In a measurement of this sort, the fission product activities of foils 1, 6, 7, and 12 are proportional to the local fission density in the fuel rod. The fission product activities of foils 3 and 4 are proportional to the fission density in the depleted uranium foil. The 0.0007-in. thick aluminum foils used here are commonly called "catcher foils." Since the range of fission products in aluminum is approximately 0.0004 in.,[5] one catcher is sufficient to stop the fission products emitted by the uranium with which it is in contact.

The activities of foils 9 and 10 give the background which must be subtracted from all other foil count rates. These decay rates are caused by the thermal activation of impurities, and by n, $2n$ and n, α reactions in the aluminum. Actually, the two foil sandwiches were not located at the same height in the lattice, and so were not exposed to the same neutron flux levels. Therefore before the background correction given by foils 9 and 10 could be applied, it had to be multiplied by the ratio of flux densities at the heights of the two sandwiches.

The determination of F from the foil-counting was carried out as follows. Catcher foils 1, 3, 4, 6, 9, and 10 were counted simultaneously, being rotated about among the counters so that each foil was counted the same number of times in each counter. At the same time, foils 7 and 12 were counted in a separate pair of counters, giving a decay curve for the fission products. This curve was used to correct the activities of the catcher foils back to the beginning of the foil-counting, called time $t = 0$. (Normally foil-counting began 30 minutes after the end of the exposure in the exponential lattice; this permitted the short-lived activity from radiative capture in the aluminum to die out.)

The backgrounds as determined from foils 9 and 10 were subtracted from the catcher foil activities, and the activities of foils 1, 3, 4, and 6 were corrected to $t = 0$ by means of the decay curve measured with foils 7 and 12. The corrected activities for each foil were then summed for all counters. Finally, these sums were added for foils 1 and 6 to obtain a quantity proportional to the fission density in the fuel rod, and were summed for foils 3 and 4 to get a number proportional to the fission density in the depleted uranium. This latter number when corrected for the slightly different atomic densities in the depleted uranium and the fuel rod was proportional to the fission density in U^{238} in the rod. Thus

$$\begin{aligned} f_{25} &= C_1(R_1 - R_2) \\ f_{28} &= C_2 R_2 \end{aligned} \qquad (32)$$

where R_1 and R_2 are respectively the final corrected decay rates for foils 1, 6, and for foils 3, 4, the latter corrected for U^{238} density. f_{25} and f_{28} are respectively the fission densities locally in the U^{235} and the U^{238} in the fuel rod. C_1 and C_2 are constants of proportionality, which would be equal except that the decay

curves of fission products from U^{238} and U^{235} are not precisely the same. Then from Equation 32,

$$\frac{f_{28}}{f_{25}} = F = K\frac{R_2}{R_1 - R_2} \tag{33}$$

with

$$K = C_2/C_1 \tag{34}$$

The quantity K depends upon the exposure time, and the period of time during which counting took place. A separate calibration experiment was necessary to determine it.

A "double" fission chamber was constructed, having the design shown in Fig. 9. The cathode of each half of the chamber had on it a carefully weighed amount of uranyl nitrate, deposited in solution and then dried. One cathode was sensitized with a sample of the depleted uranium used in the fast effect measurements; the other had natural uranium. In each case the layer of uranyl nitrate was thin compared with the path length of fission products, so that each fission on a cathode led to a count in its half of the chamber.

A sandwich of aluminum catcher foils with depleted and normal uranium foils was placed in the chamber; the arrangement of this sandwich is shown in Fig. 10.

The chamber was then exposed to a mixture of fast and thermal neutrons of approximately the same character as that observed in one of the exponential assemblies. The fission rates in the two halves of the fission chamber were recorded, and the catcher foils were afterwards removed and counted in the same way as for a fast effect measurement. The ratio of observed count rates led to the ratio of U^{235} fission rates to U^{238} fission rates in the flux used, and the catcher foil activities led to the ratio of associated decay rates.

The equivalence of the two halves of the fission chamber was tested by interchanging the cathodes.

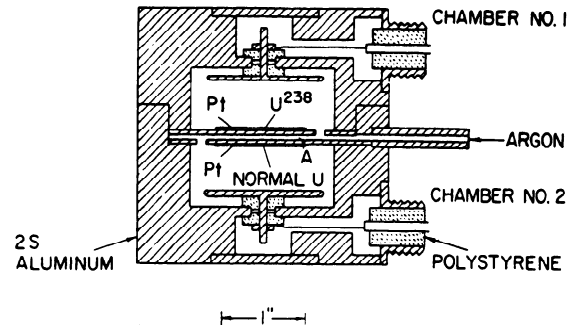

Figure 9. Fission chamber used for fast effect calibration experiment

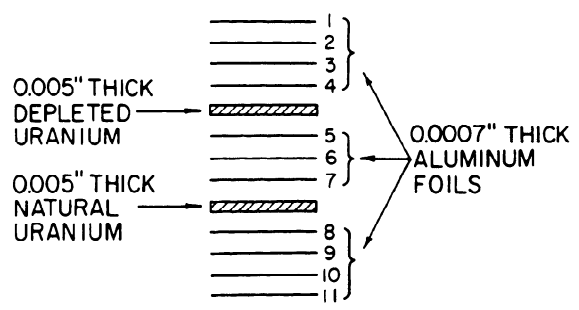

Figure 10. Schematic of uranium-aluminum sandwich used in fast-effect calibration experiment

The subsequent change in ratio of count rates was 0.3%, which was below the probable error from the counting statistics.

As a result of this calibration experiment, it was determined that for the exposure times and counters used,

$$K = 1.06 \pm 0.01 \tag{35}$$

Then from combining Equations 31, 33, 35,

$$\epsilon - 1 = 0.590\frac{R_2}{R_1 - R_2} \tag{36}$$

The values of ϵ determined in these experiments were:

$\frac{Vol.\ water}{Vol.\ uranium}$	ϵ
1.334	1.081 ± 0.005
3.834	1.051 ± 0.004

The probable error includes only counting statistics, and not the uncertainty in ν_{28}.

These values should be compared with a measurement of ϵ for a single 0.750-in. diameter uranium rod, exposed in the reflector of the Brookhaven graphite pile. There it was found that $\epsilon = 1.019 \pm 0.003$. It is apparent that the interaction fast-effect, caused by high energy neutrons from neighboring rods, is considerably greater than that of a single rod. This feature is of course directly caused by the extremely close spacing of fuel rods which the use of a light water moderator demands.

The existence of abnormally large values of ϵ in water, uranium rod lattices was predicted at an early stage in the Manhattan Project by L. Szilard.

Thermal Utilization

The values of the migration area reported earlier were obtained with calculated values of f.[2] Since there was some question about the theoretical methods used to obtain these, a direct measurement of the thermal utilization seemed useful. These measurements were also motivated, of course, by the desire to test experimentally the methods used to calculate k_∞ through the four-factor formula.

The method used was the direct one of determining the thermal neutron flux distribution in a single lattice cell, both in the fuel rod and the moderator water. The flux curves in the water and the uranium were then used to provide the respective flux averages $\bar{\varphi}_w$ and $\bar{\varphi}_u$; the thermal utilization was finally derived as

$$f = \frac{\Sigma_u{}^a\bar{\varphi}_u}{\Sigma_u{}^a\bar{\varphi}_u + \Sigma_w{}^a\bar{\varphi}_w} \tag{37}$$

with $\Sigma_u{}^a$ and $\Sigma_w{}^a$ the respective macroscopic absorption cross sections of uranium and water, averaged over a Maxwell energy distribution with the most probable velocity 2200 m/sec. The values of f obtained this way were uncertain by an amount depending on the cross-sections used, and incidentally on the mean neutron temperature. The influence of neutron temperature is however only very slight.

The flux distributions were found with foils about 1.5 millimeters in diameter and 0.25 millimeters thick,

and consisting of methyl methacrylate plastic and dysprosium oxide. The foils were punched out of a small sheet made in a hot press from a powder mixture of the two substances. Dysprosium was considered an ideal detector, because it makes cadmium difference measurements unnecessary. In fact, the cadmium ratio of a dysprosium foil in the water lattice was greater than 100.

To find the flux distribution in the uranium, it was necessary in effect to imbed the foils in a fuel rod. This was accomplished by cutting a rod in two crosswise, and machining small holes in one of the surfaces so exposed. With foils placed in these holes, the rod was then put back together and placed in the neutron flux in the exponential assembly.

The foils in the water were placed in a piece of methyl methacrylate inserted horizontally into the lattice. The two sets of detectors (in the water and in the uranium) were not placed at the same elevation. However, the difference in height was measured to within 0.1 mm and the known relaxation length for the vertical decay of the neutron flux was used to correct for the error so introduced. The undertainty produced by this correction was at most about 0.2%, and was therefore neglected.

Each foil used was counted to high statistics in each of six end-window β-ray counters. The relative activity was then obtained by summing over the observed activities (corrected for decay, etc.); thus there was no need to intercalibrate counters.

In all, six complete intracell flux traverse measurements were done; one in the 1.334, one in the 1.834, two in the 2.834, and two in the 3.834 volume ratio lattices. The experimental values of f so obtained are given in Table V; they are also plotted in Fig. 11.

Figure 11 also shows for comparison the curve of thermal utilization vs volume ratio. This theoretical curve was obtained by a diffusion theory calculation, modified for the P^3 content of the angular flux distribution. It will be observed that the theoretical curve is consistently higher than the experimental values. This feature can also be seen in terms of the actual flux distributions measured in the fuel and the moderator. The measured flux dip in the fuel was considerably greater than the calculated dip, and the measured flux rise in the moderator was considerably higher than was calculated.

Both the measurements and the calculations are still in doubt here, for reasons which will be discussed in the section on 0.600-in. diameter rods.

Table V. Experimental Values of Thermal Utilization, 0.750-in. Diameter, 1.027% Enriched Uranium Rods in Light Water

$\frac{\text{Volume water}}{\text{Volume uranium}}$	f
1.334	0.910
1.834	0.871
2.834	0.819
3.834	0.755

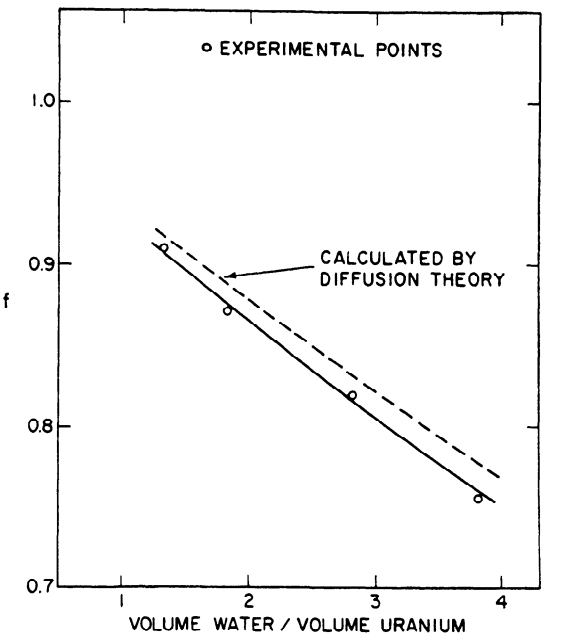

Figure 11. Thermal utilization of 0.750+ inch diameter, 1.027% enriched uranium rods in light water

Also in the section on 0.600-in. diameter rods there will be given reasons for believing that the migration area analysis using calculated values of f is in any case correct. Therefore the values of M^2 in Table IV and Fig. 7 are considered to be reliable.

BROOKHAVEN MEASUREMENTS WITH 0.600-IN. DIAMETER RODS

After the completion at Brookhaven of the exponential experiments with 0.750-in. diameter rods, just discussed, a somewhat expanded program of water moderated lattice studies was begun. This work is still under way. It involves the assembling of forty-five different lattices with all combinations of the following variables:

Rod diameter: 0.600, 0.387, 0.250 in.
Uranium enrichment: 1.299%, 1.143%, 1.027% (by weight)
$\frac{\text{Volume water}}{\text{Volume uranium}}$: 1, 1.5, 2, 3, 4

For each lattice, measurements are being made of the buckling, the migration area, the thermal utilization and the fast effect. In addition, there are some measurements of the neutron temperature and the epithermal fission fraction, and the resonance escape probability is being studied (as yet with no reliable results).

This extended program of water-uranium lattice studies has been prompted by a growing interest in the usefulness of the system for producing electrical power. In particular, the Pressurized Water Reactor under construction by the Westinghouse Atomic Power Division, to be the first large scale American nuclear power plant, is to have a design somewhat similar to that studied in the Brookhaven experiments. This fact has led to complementary programs of

experiments by the Brookhaven and Westinghouse groups, with the same fuel rods being used in turn by the two organizations at their respective sites. A discussion of the Westinghouse experiments[6] is given elsewhere in these proceedings.

Buckling

In addition to simply obtaining values of B^2, it was considered advisable to devote a fraction of the reactivity measurements to a study of the validity of the way of measuring the buckling discussed earlier. The major point in question was the assumed constancy of B^2 and λ over a wide range of numbers of fuel rods loaded. Two-group calculations by K. Puechl and a similar but more extensive study by J. Chernick indicated that the reflector savings should decrease slightly with increasing radius of core loading; this effect would cause the values of B^2 derived from the experiments to be about 3% too low.

The experimental study of the validity of the method was undertaken in four separate ways. First, the number of fuel rod loadings at which relaxation lengths were measured was increased. This permitted separate evaluations of B^2 and λ from experiments relatively near to and relatively far from critical. Thus it could be seen whether the measurement method was internally consistent in predicting the same critical masses at very different levels of reactivity. Second, a way of carrying out the more usual form of exponential experiment was found. In other words, the radial and axial buckling were found from measurements of the radial and axial flux distributions at a single high value of fuel rod loadings. One lattice (1.143% enriched, 1.5 volume ratio) was subjected to measurements of B^2 and λ by both methods. Third, a lattice whose critical mass was found by the method in question was loaded to a moderately high reactivity and the critical mass was found from an extrapolation of the observed multiplication of the spontaneous fission neutrons. Fourth, lattices measured by all these methods were taken to critical by the Westinghouse group. As a result of these four separate tests, the procedure for measuring buckling which has been discussed in connection with the 0.750-inch diameter rod lattices was shown to lead to remarkably good results.

For purposes of simplifying, the procedure discussed earlier will be called method I. To recapitulate, it consists in measuring the relaxation length L for several values of the loaded core radius R, and fitting this observed variation of L with R to a single pair of values of B^2 and λ. The procedure of finding B^2 and λ from radial and axial flux traverses will be called method II.

Method I was used to find the buckling and reflector savings of nine of the fifteen assemblies with 0.600-in. diameter rods. With each lattice, values of the vertical relaxation length were found for twenty-two different core radii. The buckling and reflector savings were determined from the top eleven and bottom eleven loadings separately, and then from all twenty-two relaxation lengths taken together.

All relaxation length measurements, except those for the 1:1 water-to-uranium volume ratio, 1.143% enrichment lattice, were made with indium foils, 0.220 in. in diameter and 0.005 in. thick. These foils were centered in a lattice cell, and oriented vertically so as to equalize the fluxes incident on opposite faces.

The foil holders were small rods of methyl methacrylate plastic, with holes milled in to contain the foils. Tight-fitting plastic plugs were inserted in the holes over the foils, keeping them secure.

The lattices with equal volumes of uranium and water were so compact that there was not enough room between fuel rods to insert the foil-holder rod. With the 1:1, 1.299% enrichment lattice, a fuel channel was therefore left empty, and the foil rod was inserted in this position. The effect of omitting the rod was then determined separately, and the actual loading was corrected to an effective one.

The same lattice with 1.143% enriched rod was treated somewhat differently. The foils used in the relaxation length measurement were exposed in a fuel rod cut into sections approximately 8 cm long. The foil diameters were the same as those of the fuel rod (0.600 in.).

Approximately 2.3 tons of each enrichment of uranium were available for these measurements, and so the experiments could be done with considerably more reactivity than was available with the 0.750-inch diameter rods. Thus in many cases the vertical relaxation length was long enough to require corrections for influence of the top boundary of the assemblies on the flux distributions.

The effect of a top boundary is to cause axial flux distributions to have the form

$$\varphi = C \sinh\left[\frac{z_0 - z}{L}\right] \tag{38}$$

where C is a constant, L is the relaxation length, and z_0 is the height at which the flux extrapolates to zero. To find L, then, one should fit the measured flux distributions to Equation 38.

Actually, end effect corrections were made as follows. z_0 was found at a high rod-loading from a least squares fit of a sinh function to a many-point axial distribution. In subsequent measurements of relaxation lengths, each saturated foil activity was multiplied by the factor

$$\frac{\exp\left[\frac{z_0 - z}{L_0}\right]}{\sinh\left[\frac{z_0 - z}{L_0}\right]} \tag{39}$$

with L_0 an approximate value of L. The corrected flux values were fitted to an exponential; this fit provided a second approximation L_1 to L. New correction factors were obtained by replacing L_0 in Equation 39 by L_1, and the calculation was recycled, the procedure being repeated until it converged. This way of correcting for end effects was considerably simpler than fitting a sinh function to each set of flux values in a relaxation length measurement.

With each of the nine lattices to which method I was applied, twenty-two values of the vertical relaxation were found at different core radii. The relaxation lengths were divided into two groups; those with the eleven largest numbers of fuel rods loaded, and those with the eleven smallest numbers of rods. These two sets of data were analyzed separately for the buckling and reflector savings, and then another determination was provided by combining all twenty-two relaxation lengths in a single calculation. The results of these analyses are shown in Tables VI and VII. A comparison of B^2 as obtained from the highest numbers and lowest numbers of fuel rods loaded shows no apparent trend of one set being higher consistently than the other. Therefore method I seems to lead to results which are at least internally consistent.

Method II was used to find the buckling and reflector savings of seven of the lattices. In these cases the assembly was loaded to a k_{eff} of about 0.97, and measurements were made of the radial and axial thermal neutron distributions. It was assumed that far enough from the source and the boundaries, the flux had the macroscopic (cell-to-cell) space variation

$$\varphi = AJ_0\left(\frac{2.4048}{R+\lambda}r\right)\sinh\left(\frac{z_0 - z}{L}\right) \qquad (40)$$

with R defined by Equation 7 and A a constant. Least squares fits of the radial flux to the Bessel function led to values of λ, and least squares fits of the axial flux to the sinh function gave values of L. B^2 was then found from

$$B^2 = \left(\frac{2.4048}{R+\lambda}\right)^2 - \frac{1}{L^2} \qquad (41)$$

Axial flux distributions were determined as discussed earlier. The radial flux distributions were obtained with indium foils placed in a fuel rod. Two such rods were used with each lattice. One was used to expose foils at successive positions on a radius, while simultaneously foils were exposed in the other rod at a fixed location. The foils in the fixed rod served as monitors, registering any changes which may have occurred in the source strength. The ratios of saturated activities of the foils in the first rod to those in the second were therefore the relative fluxes at different points on a radius.

Since it was not possible to load the fuel rods exactly in the shape of a cylinder, the flux distribution given by Equation 40 was not strictly correct. Therefore flux measurements were made along six separate radii, as shown in Fig. 12. Three of these were diagonals of the basic hexagonal pattern in the lattices; these are called the "straight" radials. The other three were not straight lines, but were formed by zig-zagging out along paths intermediate between the straight radial lines.

The three flux positions farthest from the center on each radial traverse showed the characteristic be-

Table VI. B^2 for Lattices of 0.600-in. Diameter, Slightly Enriched Uranium Rods in Light Water, as Found by Method I

Volume water / Volume uranium	U^{235} weight per cent	$B^2(cm^{-2} \times 10^4)$		
		High 11 loadings	*Low 11 loadings*	*All loadings*
1	1.299	31.08 ± 1.24	34.71 ± 1.18	32.11 ± 0.54
1.5	1.299	53.52 ± 1.25	52.31 ± 0.78	51.87 ± 0.50
2	1.299	60.88 ± 0.78	60.58 ± 1.50	61.08 ± 0.32
3	1.299	61.16 ± 0.43	58.84 ± 0.94	60.99 ± 0.26
4	1.299	49.82 ± 0.75	49.17 ± 0.69	50.28 ± 0.27
1.5	1.143	40.15 ± 0.55	40.14 ± 0.68	40.23 ± 0.30
2	1.143	47.53 ± 1.10	48.15 ± 0.63	48.22 ± 0.31
3	1.143	46.82 ± 0.81	47.52 ± 0.96	47.12 ± 0.33
4	1.143	35.97 ± 0.53	35.96 ± 0.37	36.03 ± 0.16

Table VII. Reflector Savings of Lattices of 0.600-in. Diameter, Slightly Enriched Uranium Rods in Light Water, as Found by Method I

Volume water / Volume uranium	U^{235} weight per cent	λ (cm)		
		High 11 loadings	*Low 11 loadings*	*All loadings*
1	1.299	8.13 ± 0.27	7.60 ± 0.16	7.94 ± 0.10
1.5	1.299	7.02 ± 0.32	7.38 ± 0.16	7.44 ± 0.10
2	1.299	7.08 ± 0.16	7.11 ± 0.21	7.04 ± 0.06
3	1.299	6.67 ± 0.09	6.94 ± 0.12	6.70 ± 0.05
4	1.299	6.80 ± 0.21	6.82 ± 0.12	6.64 ± 0.07
1.5	1.143	7.52 ± 0.16	7.51 ± 0.12	7.50 ± 0.08
2	1.143	7.29 ± 0.30	7.11 ± 0.12	7.10 ± 0.25
3	1.143	6.76 ± 0.25	6.66 ± 0.18	6.68 ± 0.10
4	1.143	6.46 ± 0.06	6.46 ± 0.23	6.45 ± 0.10

havior of the flux rise near the reflector. These points were therefore discarded in the analysis, and the least squares fits which led to the experimental values of the reflector savings λ were based only on the remaining portion of the flux curves.

With one lattice (1:1 volume ratio, 1.143% enriched rods) flux traverses were made along radii at two different heights in an attempt to discover if the reflector savings was independent of z. Within the experimental error of the measurement (about 0.2 cm in λ), no difference was observed.

The measured values of B^2 and λ are listed in Tables VIII and IX. The combined results of measurements of buckling and reflector savings obtained by both experimental methods are shown in Figs. 13 and 14, for the three fuel enrichments.

The buckling and reflector savings of one assembly (1.143% enriched, 1.5 volume ratio) were obtained with both methods I and II. The results agree to within their indicated probable errors.

One determination of the critical mass was made by extrapolating to infinity the multiplication of spontaneous fission neutrons. The lattice used consisted of 1.143% enriched rods with a water-to-uranium volume ratio of 2. This assembly was loaded to a moderately high neutron multiplication, with appropriate flux monitors, safety rod, and emergency shut-down equipment, and with high flux levels maintained by a Po-Be source. At the maximum loading, the source was removed. A small BF_3 counter was inserted near the central fuel rod, and the count rates were measured. Rods were then removed from the periphery, so that count rates could be measured at several smaller rod loadings.

Table VIII. Buckling of Lattices of 0.600-in. Diameter, Slightly Enriched Uranium Rods in Light Water, Measured by Method II

Volume water / Volume uranium	U^{235} by weight	B^2 ($cm^{-2} \times 10^4$)
1	1.143%	21.19 ± 0.23
1.5	1.143	39.87 ± 0.56
1	1.027	9.66 ± 0.52
1.5	1.027	29.60 ± 0.70
2	1.027	35.83 ± 0.47
3	1.027	32.93 ± 0.34
4	1.027	21.01 ± 0.29

Table IX. Reflector Savings of Lattices of 0.600-in. Diameter, Slightly Enriched Uranium Rods in Light Water, Measured by Method II

Volume water / Volume uranium	U^{235} by weight	(cm)
1	1.143%	8.15 ± 0.13
1.5	1.143	7.62 ± 0.18
1	1.027	8.68 ± 0.15
1.5	1.027	7.49 ± 0.23
2	1.027	7.14 ± 0.19
3	1.027	6.63 ± 0.18
4	1.027	6.43 ± 0.19

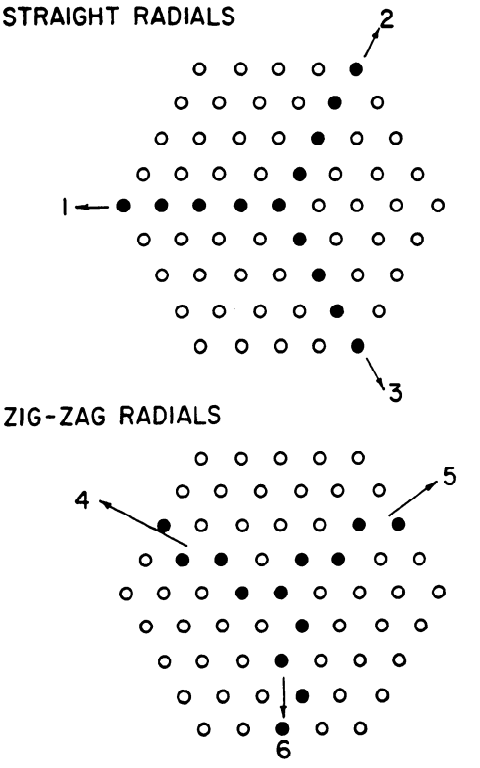

Figure 12. Lines along which radial flux distribution was measured

According to diffusion theory, the thermal neutron flux at any given point in the lattice should (sufficiently near critical) have the form

$$\varphi = \text{constant} \cdot \left\{ \left(\frac{a_0}{R+\lambda} \right)^2 - \left(\frac{a_0}{R_c+\lambda} \right)^2 \right\}^{-1} \quad (42)$$

where a_0 is the first root of the zero order Bessel function, and R_c is the critical radius. Therefore a plot of the reciprocal of the count rate against the quantity

$$\left(\frac{a_0}{R+\lambda} \right)^2 \quad (43)$$

should be a straight line, intersecting the axis at

$$\left(\frac{1}{R+\lambda} \right)^2 = \left(\frac{1}{R_c+\lambda} \right)^2 \quad (44)$$

Clearly, the solution of Equation 44 depends on the value of λ chosen. However, the measurements were done at a high enough neutron multiplication to define R_c quite well. For instance, the indicated critical mass for λ = 7.5 cm was 471 rods, while at λ = 0 cm it was 472 rods. Thus for a reasonable value of λ (the buckling measurements gave λ = 7.10 cm), the critical loading was 471 rods.

The values of B^2 and λ found in buckling measurements by method I led to a predicted critical mass of 467 rods. The difference amounts to about 0.9% in B^2, which is the same order of magnitude as the random errors in the buckling measurements.

Thus the Brookhaven measurements have been shown in three separate ways to be consistent, within their experimental errors.

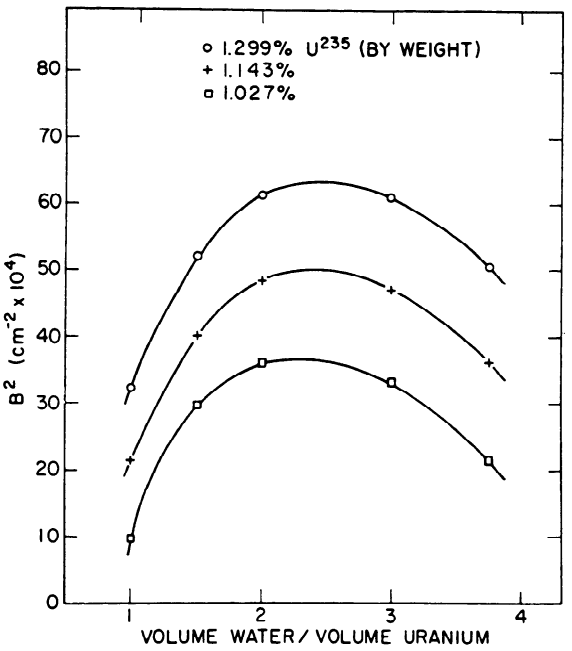

Figure 13. Experimental values of buckling, 0.600 inch diameter, slightly enriched uranium rods in light water

The Westinghouse group has taken to critical several of the lattices studied at Brookhaven.[6] The critical masses reported to date have been compared with the Brookhaven predictions in Table X. The two most accurate Brookhaven measurements of the buckling predicted critical masses which were observed within experimental error to be correct. The least accurate Brookhaven measurement predicted a critical mass which differs from the observed value a little more than the probable error.

Thus every attempt to find out how valid the Brookhaven measurements are has led to the conclusion that they are self-consistent and correct, with the cited experimental error being a good measure of the inaccuracy in predicted critical size. The probable errors in the values of B^2 and λ given in the tables were derived solely from the standard deviations from the least squares fits. Thus there seem to be no appreciable systematic errors in the experiments.

Fast Effect

The fast effect should be nearly independent of fuel enrichment in the range covered by these experiments. Therefore it was not considered necessary to find ϵ for all the lattices which were assembled. Instead, measurements were done in eight of the fifteen lattices, including all five volume ratios with the 1.299% enriched uranium and three volume ratios with the 1.143% enriched uranium. In addition, one measurement was carried out in a lattice with a water-to-uranium volume ratio of 0.1785, which was almost the smallest which could be obtained by close-packing the uranium. The fast effect for such a small volume ratio is close to that in a large solid piece of uranium.

The experimental methods discussed in connection with the 0.750-in. rod measurements were used here with no significant change. The experimental results are given in Table XI, and have been plotted as a function of the water-to-uranium volume ratio in Fig. 15. The probable errors given in Table XI are derived solely from the statistics of foil-counting, and do not include systematic errors.

R. L. Hellens, of the Westinghouse group, has pointed out that some of his calculations of the fast effect of these assemblies show that a small but appreciable correction must be applied to account for fast neutron leakage. In an assembly with a positive value of the buckling, there is a net flow outward of neutrons from each point in the core, and this feature would tend to decrease the measured value of ϵ below that calculated for an infinite lattice. Moreover, the experimental fast effect should then depend on the buckling, and hence on the fuel enrichment. The effect is not very large, and it is difficult to calculate accurately.

Unfortunately, the measurements were not accurate enough to establish the magnitude of this trend. ϵ was measured at two different fuel enrichments for three of the five volume ratios, viz: 4:1, 1.5:1, and 1:1. The latter two show a possible increase in ϵ with decreasing enrichment (hence decreasing buckling), but the observed differences are just within the experimental errors. The former shows an opposite trend—which also is zero to within experimental error. Thus no conclusions on the size of this effect could be drawn. It appears in any case to be small.

Thermal Utilization

The measurements of f which were begun with 0.750-in. diameter rods were continued with 0.600-in. rods. The method was still an experimental determina-

Table X. Comparison of Brookhaven and Westinghouse Criticality Studies with 0.600-in. Diameter Rods of 1.299% Enriched Uranium in Light Water

Volume water / Volume uranium	Critical number of rods	
	Brookhaven (predicted)	Westinghouse (observed)
1.5	486.6 ± 7.4	478
2	337.4 ± 2.9	335
3	265.6 ± 1.9	266

Table XI. Experimental Values of the Fast Effect, 0.600-in. Diameter Rods of Slightly Enriched Uranium in Light Water

Volume water / Volume uranium	% U^{235} by weight	ϵ
1	1.299	1.105 ± .002
1.5	1.299	1.072 ± .001
2	1.299	1.061 ± .001
3	1.299	1.047 ± .001
4	1.299	1.043 ± .001
1	1.143	1.109 ± .002
1.5	1.143	1.074 ± .001
4	1.143	1.042 ± .001
0.1785	1.299	1.227 ± .011

tion of the intracell flux distribution. Some of the techniques, however, were changed in the course of measurements with the smaller rods.

The greatest change was introduced in the way foils were held in position in the moderator. When the 0.750-in. rods were being studied, the foils in the water were placed in rectangular slabs of methyl methacrylate plastic, which were inserted horizontally between the fuel rods. The foil holders used with 0.600-in. rods were small triangular-symmetried pieces such as that shown in Fig. 16; these were inserted vertically into the lattices through holes in the top fuel-rod locating plate. The geometry of the new foil holders permitted the use of a much larger number of foils in the water than before, and thus led to more detailed flux plots. Figure 16 shows a typical experimental flux plot, in this case for the 1.5:1 volume ratio, 1.143% enrichment lattice.

The foil holders used with the 4:1 and 3:1 volume ratio lattices of 1.299% enriched rods were made of 0.6 millimeter thick methyl methacrylate. Those used for all other measurements consisted of a double thickness of 0.08 millimeter thick aluminum. The two groups of measurements did not seem to agree very well. Similar measurements of f were carried out by the Westinghouse group, with methyl metha-

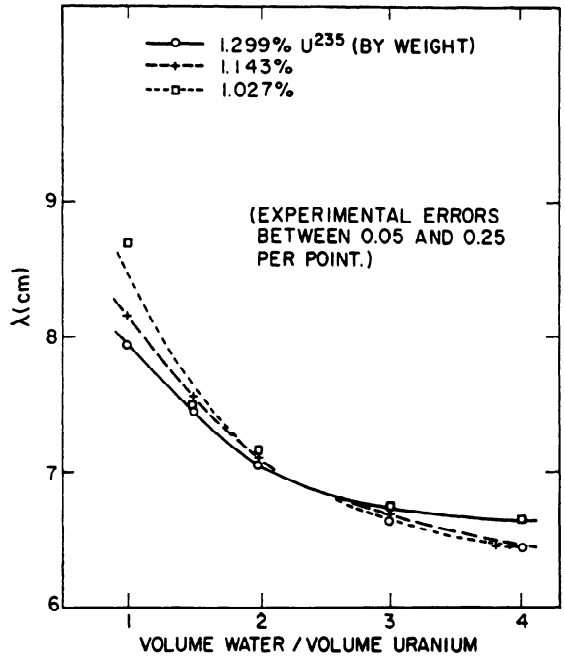

Figure 14. Reflector savings of 0.600 inch diameter, slightly enriched uranium rods in light water

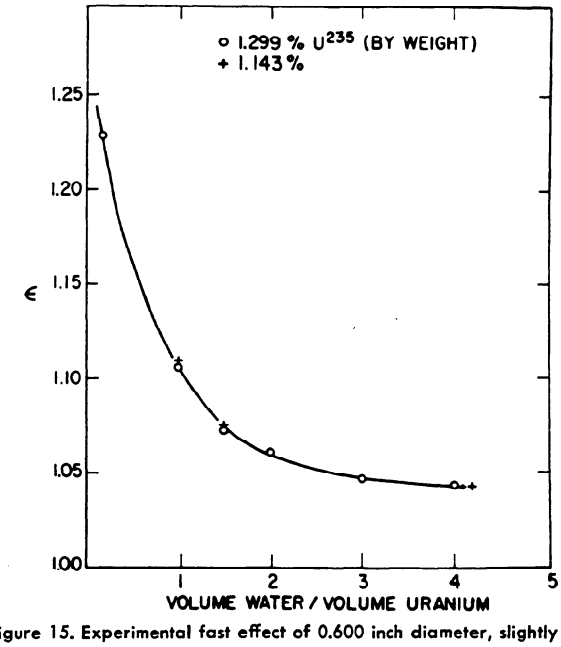

Figure 15. Experimental fast effect of 0.600 inch diameter, slightly enriched uranium rods in light water

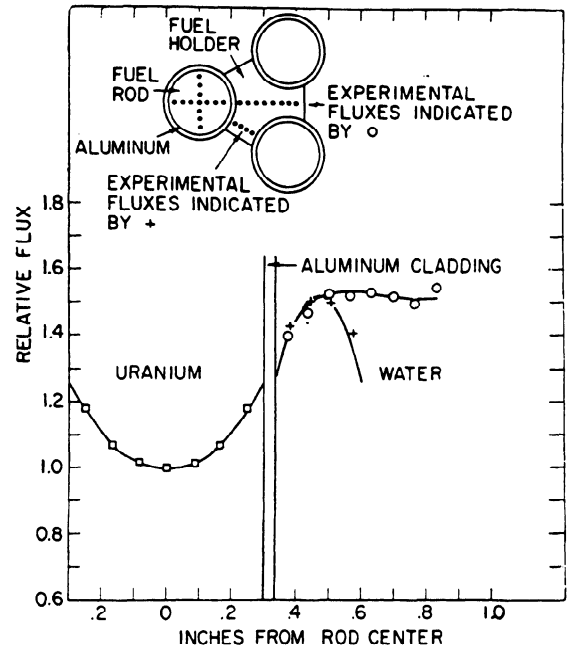

Figure 16. Experimental intracell flux traverse, 0.600 inch diameter, 1.143% enriched uranium rods in light water. Volume ratio = 1.5:1

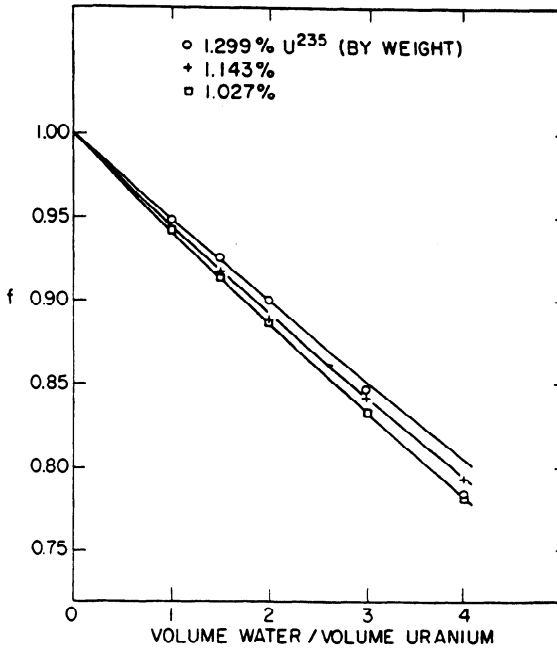

Figure 17. Measured thermal utilization, 0.600 inch diameter, slightly enriched uranium rods in light water

Table XII. Experimental Values of Thermal Utilization for Slightly Enriched Uranium Rod, Light Water Moderated Lattices

Volume water / Volume uranium	% U^{235} by weight	f (measured)	f (from smooth curves)
1	1.299	0.948	0.949
1.5	1.299	0.925	0.924
2	1.299	0.900	0.899
3	1.299	0.846	0.851
4	1.299	0.784	0.804
1	1.143	*	0.945
1.5	1.143	0.917	0.918
2	1.143	0.888	0.893
3	1.143	0.842	0.842
4	1.143	0.793	0.793
1	1.027	0.941	0.941
1.5	1.027	0.913	0.912
2	1.027	0.886	0.885
3	1.027	0.833	0.833
4	1.027	0.781	0.781

* Not measured.

crylate foil holders used throughout. There was a consistent tendency for measurements done with aluminum foil-holders to lead to larger values of f than those done with the plastic. The source of the disagreement is not certain; because of its existence, however, the measurements which are reported here must be considered as in doubt by an amount varying from about 0.01 in f at the 4:1 volume ratio to about 0.002 in f at the 1:1 volume ratio. This uncertainty probably will be resolved in the very near future.

The experimentally measured thermal utilizations for fourteen of the fifteen lattices are given in Table XII. Values of f taken from the smooth curve plots of Fig. 17 are also given. A measurement was not performed for one of the fifteen assemblies; this was the one with a 1:1 volume ratio and 1.143% enriched rods. The missing number can however be interpolated directly from the curves with very good accuracy, and this has been done.

Two points deviate markedly from the smooth curves in Fig. 17. These were the measurements made with plastic foil holders.

The implications of these experiments for the mean neutron temperature in the moderator are very interesting. The wider spaced lattices have values of f which apparently can be matched with P_3 calculations based on a mean neutron energy of 0.035 volts. The results with more closely spaced lattices (particularly the very tightly grouped 1:1 volume ratio) fit much higher neutron temperatures. The precise values of the temperature which lead to a good fit are not important, because of uncertainties in the methods of calculation. However, the trend toward higher temperature with closer rod spacings has been definitely observed.

Migration Area

The migration areas of these assemblies are of interest mostly because they are ingredients in the analysis of the neutron economy. With the buckling and the migration area known, k_∞ is also known.

In addition, measured values of M^2 can lead to information on the inelastic and scattering cross-sections of uranium, through comparison with calculated values. This course has to be followed cautiously, however; there are reasons why the measured and calculated values could differ and yet both still be right. This point will be discussed later after the results of the experiments have been presented.

Migration area measurements with the 0.600-in. diameter rods were done in two different but related ways. Briefly, these were

(1) The moderator water was poisoned with B_2O_3, and the thermal utilization and buckling were measured at several such concentrations. This was the measurement used with 0.750-in. rods also, and was discussed earlier in connection with those experiments. In the following it will be called "the poison method."

(2) Values of thermal utilization and buckling were measured as a function of fuel rod enrichment. This will be called "the enrichment method."

Theoretically, both kinds of method depend on use of the critical equation:

$$\epsilon p f \eta \mathfrak{L} = 1 \tag{45}$$

where $\mathfrak{L}$ is the fractional neutron non-leakage in a single fission cycle. According to one-group diffusion theory,

$$\mathfrak{L} = (1 + M^2B^2)^{-1} \tag{46}$$

On the other hand, the expression derived from age theory (continuous slowing-down is)

$$\mathfrak{L} = \exp[-\tau B^2](1 + L_{th}^2B^2)^{-1} \tag{47}$$

For the assemblies studied here, B^2 was at most about 6×10^{-3}, and L^2 was about 4 cm.^{2} Thus at most

$$B^2L_{th}^2 \sim 2.4 \times 10^{-2} \tag{48}$$

Therefore

$$(1 + B^2L_{th}^2)^{-1} \sim \exp[-B^2L_{th}^2]$$

$$\mathfrak{L} = \exp[-B^2\tau](1 + L_{th}^2B^2)^{-1} \sim \exp[-M^2B^2] \tag{48a}$$

At the outset, one is faced with having to choose between describing the leakage by means of Equations 46 and 48a. There are reasonable *a priori* arguments for each. Equation 48a is the limiting expression for multigroup diffusion theory; on the other hand, the thermal flux distributions arising from the slowing-down of neutrons in water can be fitted reasonably well to a one-group diffusion kernel. The latter however leads to Equation 46. Since grounds for a choice between the two non-leakage kernels were not apparent at the beginning, analysis of the experiments was done starting from both assumptions. This procedure eventually led to a choice of Equation 48a as the preferred form for the non-leakage.

The "poison method" of finding M^2 consists in adding B_2O_3 to the moderator water, and then measuring B^2 and f. The presence of the boron does not affect the values of ϵ, p, η, and M^2 very much, and so from Equations 45 and 46,

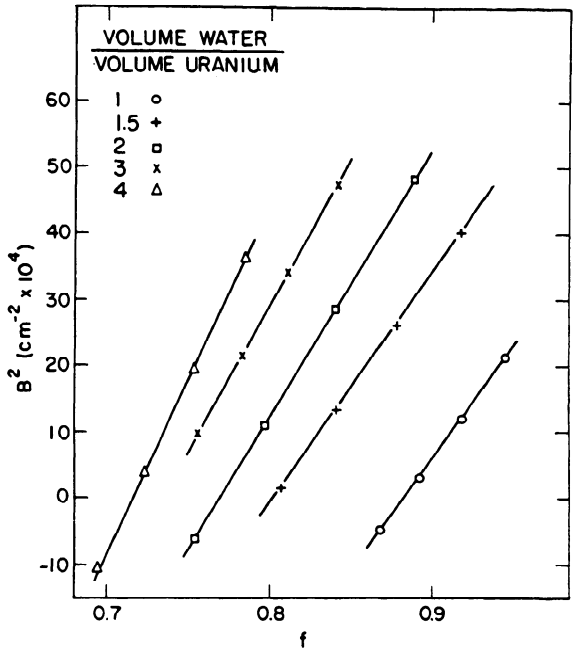

Figure 18. f vs B^2 for boron poisoned lattices of 0.600 inch diameter, 1.143% enriched uranium rods in light water

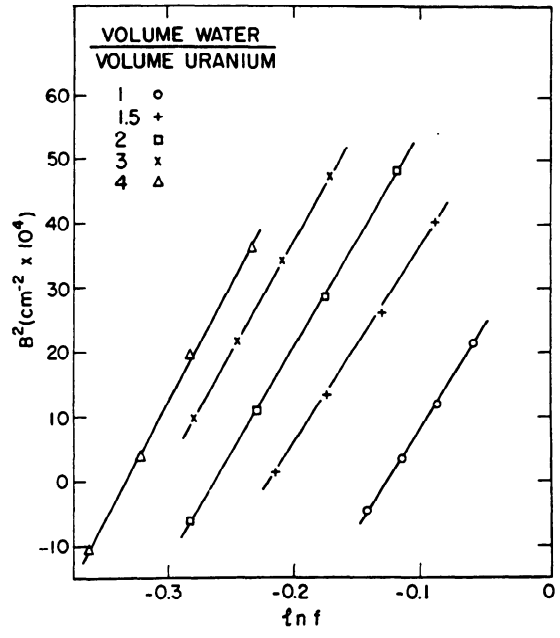

Figure 19. ln f vs B^2 for boron poisoned lattices of 0.600 inch diameter, 1.143% enriched uranium rods in light water

$$\epsilon p\eta \left[\frac{df}{d(B^2)}\right] = M^2$$

Also from Equations 45 and 46,

$$\epsilon p\eta[f]_{B^2=0} = 1$$

Therefore

$$M^2 = \frac{1}{[f]_{B^2=0}} \frac{df}{d(B^2)} \tag{49}$$

One-group theory therefore relates the measured values of f and B^2 linearly, and the migration area is the slope of the straight line divided by its intercept.

An analysis of the experiment with the age theory kernel starts from recasting Equations 45 and 48a in the form

$$\ln(\epsilon p\eta) + \ln f = M^2B^2$$

In this case it is $\ln f$ which is linearly related to B^2, and M^2 is calculated as

$$M^2 = \frac{d(\ln f)}{dB^2} \tag{50}$$

To obtain M^2 by the "enrichment method" one must first notice that η and f are simultaneously changed by varying the U^{235} content of the fuel. Including this in the analysis leads to two expressions similar to Equations 49 and 50. If one-group theory is assumed,

$$M^2 = \left[\frac{1}{(\eta f)_{B^2=0}}\right]\left[\frac{d(\eta f)}{d(B^2)}\right] \tag{51}$$

Or assuming the validity of the age-theory kernel,

$$M^2 = \frac{d(\ln \eta f)}{d(B^2)} \tag{52}$$

In the poison measurements, boron was added to the water as boric acid crystals. Care was taken to assure complete solution of the salt, the mixture being agitated for several hours with a mechanical stirrer. The boron concentrations were then found by chemical analysis. In each case at least two quantitative boron determinations were made, and these generally agreed to within better than 0.5%.

The poison method was used with 1.143% enriched rods to find the migration areas of all five of the lattices which were assembled. The thermal utilizations and bucklings were measured by the methods discussed previously. Figures 18 and 19 show the experimental results, with B^2 plotted against f and $\ln f$, respectively. Since the two variables were not necessarily measured with the same boron concentrations in every case, the points upon which the curves of Figs. 18 and 19 are based are interpolations. The curves show equally good fits of the measured points to straight lines.

The curves of Figs. 18 and 19 were subjected to least squares fitting, to provide the slopes, the intercepts, and hence the migration areas. The results of the analysis are given in Table XIII. The values of k_∞ calculated from the measured bucklings and migration areas are shown in Table XIV. It is apparent that though the sets of values of M^2 obtained using the two critical equations differ noticeably. the values of k_∞ match quite well. Thus the latter quantity may be considered as well-determined.

The measurements of f and R^2 upon which the enrichment method of determining M^2 depends have already been discussed. In carrying through the analysis with Equations 51 and 52, however, it is necessary to use calculated values of η. Consistency in the choices for the different enrichments is easily

obtained; one begins with a value of η for natural uranium, writing it as

$$\eta_{nat} = \frac{\eta_{25}}{1 + N\sigma_{28}^a/\sigma_{25}^a}$$

with η_{25} the (preassumed) value for pure U^{235}, N the atomic ratio of U^{238} to U^{235} in natural uranium, and σ_{28}, σ_{25} respectively the mean thermal cross sections of U^{238} and U^{235}. These choices of η_{nat} and η_{25} then determine a value for the ratio

$$\frac{\sigma_{28}^a}{\sigma_{25}^a}$$

and the enrichment dependence of η is obtained from the variation of N.

The best values of present cross sections give

$$\eta_{nat} = 1.327 \pm 0.015 \qquad (53)$$

The analysis of various reactor experiments has not been any more successful in fixing this number; in the literature one can find values ranging from 1.28 to 1.35. On the basis of the evidence available, the value in Equation 53 seems to be the best yet determined, and it was chosen for analyzing the migration area measurements.

The one-group migration areas obtained using Equation 51 were found from least-squares fits of (ηf) against B^2. The age-theory values obtained from the use of Equation 52 were obtained from least-squares fits of straight lines to the variation of ln (ηf) against B^2. The results are given in Table XV. The probable errors assigned are compounded from the probable errors in B^2 and f and the uncertainty in η.

Additional analyses of the poison method were made with measured values of B^2 and with values of f which were calculated by two different schemes. One set of calculated thermal utilizations was obtained from ordinary diffusion theory; another was found from P_3 calculations. The calculated values of f did not match the experimental values very well. Nevertheless, the deduced migration areas were not changed appreciably from the values obtained with the use of purely experimental data. Therefore it is concluded that uncertainties in thermal utilization measurements do not affect the migration areas which have been obtained.

Table XVI, giving the "enrichment method" values of k_∞ for the 1.143% enriched rods, can now be compared with Table XIV, which is based on migration areas obtained from the poison measurements. It is apparent that the two sets of values of k_∞ agree moderately well (except for the 4:1 lattice) if the age theory analysis is used, but seem irreconcilable if the one-group critical equation is used. This result seems to establish quite definitely that the more correct critical equation is, as obtained from Equations 45 and 48a

$$\epsilon p f \eta = \exp[M^2 B^2] \qquad (54)$$

Equation 50 is a special case of Equation 52, where η is not varied. Therefore all of the measured values of f and B^2 for the different boron poisonings and fuel rod enrichments can be analyzed at once with the latter. Best values of M^2 have been obtained in this way; the results have been listed in Table XVII, and are plotted in Fig. 20.

The reasons for the large difference between these results and those obtained at Oak Ridge has been discussed in connection with the 0.750-in. diameter rod measurements. There is also a noticeable difference between the experimental results with 0.600 and 0.750-in. rods. The migration areas with the smaller rods are larger, a feature which can readily be ascribed to the greater first-flight inelastic scattering probability when larger rods are used. Also, the migration areas of the smaller rods show a definite tendency to

Table XIII. Migration Areas of Lattices According to the Poison Method. 0.600-in. Diameter Rods of 1.143% Enriched Uranium in Light Water

Volume water / Volume uranium	M^2 (cm^2) from one-group equation	M^2 (cm^2) from age theory equation
1	33.66 ± 1.08	32.78 ± 1.08
1.5	35.55 ± 1.26	33.15 ± 0.88
2	32.19 ± 0.77	30.21 ± 0.69
3	30.91 ± 1.26	28.38 ± 1.07
4	28.75 ± 1.17	27.72 ± 1.08

Table XIV. k_∞ for 1.143% Enriched, 0.600-in. Diameter Uranium Rods in Light Water, (Values Based on Measured B^2 and on M^2 from Table XIII)

Volume water / Volume uranium	k_∞ from one-group analysis	k_∞ from age theory analysis
1	1.071 ± 0.003	1.072 ± 0.003
1.5	1.143 ± 0.005	1.142 ± 0.005
2	1.155 ± 0.005	1.157 ± 0.004
3	1.146 ± 0.007	1.143 ± 0.006
4	1.104 ± 0.005	1.105 ± 0.005

Table XV. Migration Areas of 0.600-in. Diameter Rods of Slightly Enriched Uranium in Light Water. Obtained by Enrichment Method

Volume water / Volume uranium	M^2 from one-group analysis (cm^2)	M^2 from age theory analysis (cm^2)
1	34.34 ± 1.63	32.29 ± 1.39
1.5	39.91 ± 2.09	34.59 ± 1.64
2	36.94 ± 1.43	31.57 ± 1.15
3	36.08 ± 1.22	30.62 ± 1.30
4	36.23 ± 1.28	31.91 ± 1.33

Table XVI. k_∞ for 1.143% Enriched, 0.600-in. Diameter Uranium Rods in Light Water (Values Based on Measured B^2 and on M^2 from Table XV)

Volume water / Volume uranium	k_∞ from one-group analysis	k_∞ from age theory analysis
1	1.073 ± 0.004	1.071 ± 0.003
1.5	1.161 ± 0.009	1.149 ± 0.007
2	1.178 ± 0.007	1.164 ± 0.006
3	1.170 ± 0.006	1.155 ± 0.007
4	1.131 ± 0.005	1.122 ± 0.005

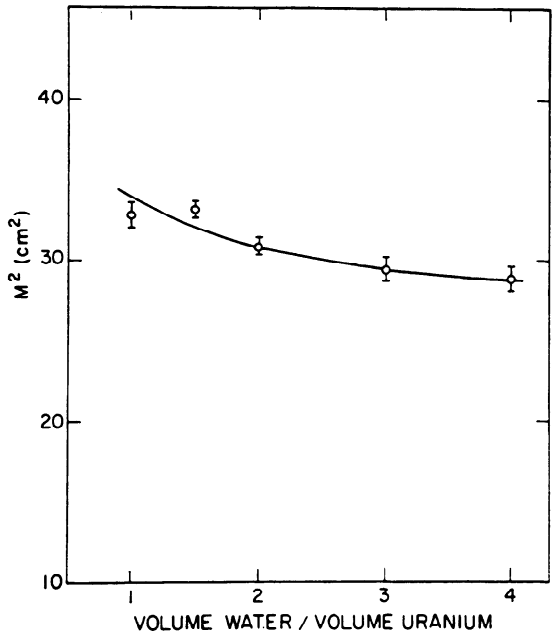

Figure 20. Best values of migration areas of fission neutrons in lattices of 0.600 inch diameter, slightly enriched uranium rods and light water

increase as the volume ratio is diminished. Such a trend seems to exist also with the 0.750-in. rod values (Table IV), but it is not outside experimental error.

As mentioned earlier, one must be careful about comparing these experimental values of M^2 with those calculated by neutron slowing-down theory. The lattices studied are strongly heterogeneous, and strictly speaking the migration areas which have been measured are defined only by the non-leakage probability (Equation 48a). It is not at all certain that these quantities are related to $\overline{r^2}$ in the same way as the migration area for homogeneous assemblies. In the past, attempts have been made to "homogenize"

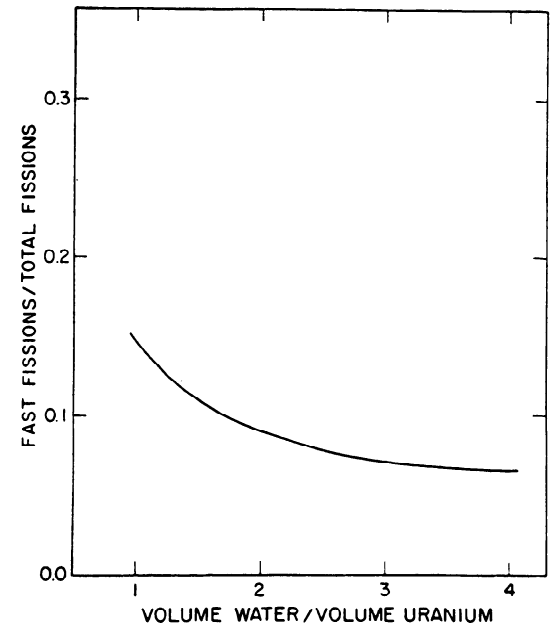

Figure 21. Experimental fast fission fraction for 0.600 inch diameter uranium rod lattices in light water

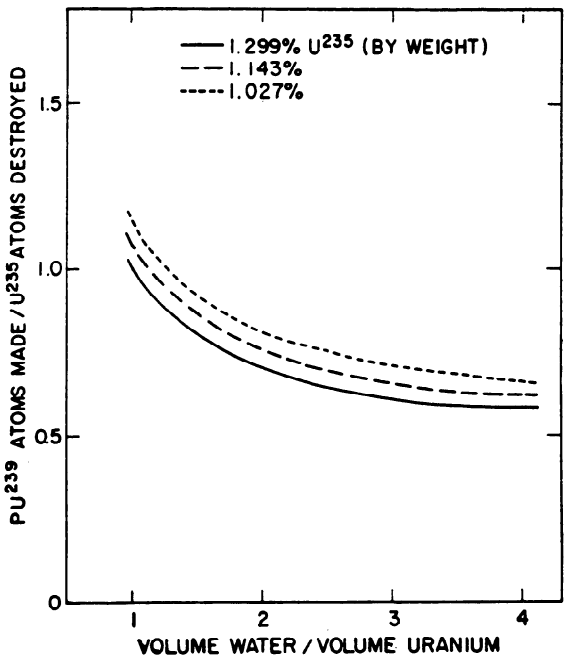

Figure 22. Conversion ratio of 0.600 inch diameter, slightly enriched uranium rod lattices in light water

heterogeneous reactor theory by defining quantities like the mean age, cross-sections, and diffusion lengths over a lattice cell, but these efforts have all depended on some simplified model of the slowing-down and diffusion processes. The most that can be said in general is that the calculated (from slowing-down theory) and measured (from leakage) values of the migration area for heterogeneous assemblies should be of the same orders of magnitude.

Neutron Temperature

The mean thermal neutron temperature of one lattice (1.143% enriched rods, 3:1 volume ratio) was found by a method resembling that used by G. B. Gavin at the Knolls Atomic Power Laboratory.[9] The experiment consisted in finding the relative effects on neutron multiplication of boron and cadmium in solution in the water moderator. The ratio of the observed danger coefficients led to the ratio of the mean cross-sections of the two poisons over the low energy neutron flux spectrum; since the two have very different cross-section curves in this energy region, a basis for estimating the neutron temperature existed.

Table XVII. Best Experimental Values of Migration Areas of Fission Neutrons in Lattices of 0.600-in. Diameter, Slightly Enriched Uranium Rods in Ordinary Water

$\frac{\text{Volume water}}{\text{Volume uranium}}$	M^2 (cm^2)
1	32.70 ± 0.77
1.5	33.05 ± 0.55
2	30.78 ± 0.44
3	29.31 ± 0.73
4	28.70 ± 0.71

The lattice was loaded to a neutron multiplication of about 500, with suitable critical assembly precautions being taken, and the count rate from spontaneous neutron multiplication was measured. A predetermined amount of boric acid solution was added to the moderator, after the equivalent volume of water had been removed, and the count rate was again measured. A measured amount of cadmium sulfate solution was then similarly added, and the count rate was recorded again.

It can be shown that the relation between the observed count rates and the small amounts of poison is

$$\frac{\frac{1}{R_2} - \frac{1}{R_1}}{\frac{1}{R_1} - \frac{1}{R_0}} = \frac{\Sigma_{Cd}}{\Sigma_B} \tag{55}$$

where the R_i are the observed count rates in the order in which they were measured, and the Σ's are the indicated macroscopic cross-sections of the poisons in the moderator. These macroscopic cross-sections were averaged over the $1/v$ energy dependence of boron and the characteristic resonance curve of cadmium, with the Maxwell energy distribution used as a weight function. It was found that the experimental results could be matched with a neutron temperature of 304° ± 17°K, where the probable error contains the statistical inaccuracies of the counting and the uncertainties in the experimental cross sections of boron and cadmium. Since the water at the time of the measurement was at 297°K, the mean neutron temperature was 7° ± 17° above the moderator temperature. To within experimental error, then, the low energy neutrons were in thermal equilibrium with the water.

IMPLICATIONS OF THE EXPERIMENTS

The war-time Oak Ridge experiments led to the conclusion that light water moderated natural uranium lattices could not be used as successful high flux reactor cores. They did however point up the fact that a slight increase in enrichment would make the use of a light water moderator feasible.

The Brookhaven measurements, in addition to providing experimental data on the reactivity of such slightly enriched uranium systems, have shed some interesting light on their neutron economies.

The high values of ϵ which can be obtained with the closer spaced lattices imply that a large fraction of the heat energy which would be produced in a going reactor of this type would be derived from direct burning of the U^{238}. The fast effect measurements are really determinations of the fast fission fraction; that the latter quantity is the directly measured one, and the former is only inferred. The measured fast fission fractions with 0.600-in. rods are shown in Fig. 21. Because of these large U^{238} burnouts, a power reactor of this type could be run longer periods of time before fuel replacement became necessary.

The tightly packed lattices also have a surprisingly large plutonium production rate. This can be estimated from the measured values of k_∞, ϵ, and f, and from the assumptions regarding η which were made in connection with the migration area measurements by the enrichment method. These measured and assumed components of the neutron economy permit the inferring of the resonance escape probability. These are all of the quantities necessary for estimating the plutonium production rate in the core with reasonable accuracy. The results of such an analysis are shown in Fig. 22. It is seen that for small volume ratios, the conversion ratio, defined as the number of plutonium atoms produced per atom of U^{235} destroyed, is greater than one. This feature again is a consequence of the large fast fission fraction because an appreciable amount of U^{238} is also being burned.

It should be noted that Fig. 22 shows only the conversion factor in the core. A reactor design of this type which had also a conversion blanket would produce considerably larger amounts of plutonium.

The large values of the fast effect even make thermal plutonium breeding possible, at least in principle. The experimental value of η for Pu^{239} is 2.03 ± 0.03. Normally then one would suppose that per neutron cycle in a plutonium fueled reactor only 1.03 ± 0.03 neutrons could be made available for leakage, capture in the moderator, and breeding. Actually, with an appreciable fast effect, the number of neutrons which can be used for breeding per atom of plutonium destroyed is $\eta\epsilon$, making the possibility of thermal plutonium breeding more likely.

The numbers which have been used to estimate conversion ratios apply to clean, cold reactor cores. The high temperatures in a power reactor certainly would change the nature of the curves obtained from the Brookhaven measurements, and the conversion ratios would be modified also. There would be other changes caused by fuel burnout, and small effects from fission product build-up. The long-term processes which would occur in a power reactor are complicated. Therefore trustworthy information on the effect of these high conversion and breeding rates must eventually be found from the behavior of such full scale reactor projects, particularly from the reactivity changes over long periods of time.

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