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UNION CARBIDE

OAK RIDGE Y-12 PLANT
OAK RIDGE, TENNESSEE

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Nuclear Division
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ABSTRACT

Array critical experiments have been performed with small and large enriched UO_2 units, with external moderator between units and with internal moderator in the form of an alcohol-UO_2 slurry.

Calculated multiplication factors of the experimental arrays, derived from the KENO Monte Carlo code and Hansen-Roach 16-group cross sections, were in satisfactory agreement with the observed values, i.e., $|\Delta k| < 0.01$. These experiments and calculations have demonstrated further the capability of the Hansen-Roach cross sections used in various reactor codes to predict criticality for enriched uranium.

INTRODUCTION

There has been a need for criticality data using fissile material in the chemical form in which it is made, used, transported, and stored. With the availability of Monte Carlo reactor codes, and especially KENO,[1] calculations of the multiplication factor, k, can be performed of arrays or configurations of a number of the containers in which fissile material is placed. Such containers for the fissile units in array studies can be used in the critical experiment program and the cost of preparing special materials is eliminated.

It was logical therefore to use some small cans of $U(93.15)O_2$[2] and also to repackage 360 kg of $U(93.15)O_2$ in cylindrical gallon-size tins during an inventory blending and sampling procedure for accountability. Part of this latter oxide was used to provide units containing hydrogenous moderator (alcohol) intermixed with the oxide to simulate conditions similar to wet oxide powders.

This report describes critical experiments performed with these various oxide fissile units in array studies thereby providing reference criticality data for calculational "bench mark" experiments for fissile material in the chemical form of UO_2.

EXPERIMENTAL MATERIALS

The fissile material for the units in the array studies was highly enriched UO_2. Table 1 is a summary of the properties and descriptions of the four types of UO_2 fissile units used in the experimental program. The detailed spectrochemical and other analyses of the oxide in the 18 Type II fissile units are given in Appendix A. Nine of these were used to make the Type III and IV units.

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1. G. E. Whitesides and N. F. Cross, "KENO - A Multigroup Monte Carlo Criticality Program," CTC-5, Oak Ridge Computing Technology Center (1969).
2. U(93.15) signifies uranium enriched to 93.15 wt% ^{235}U.

Table 1. Description of Fissile Units of $U(93.15)O_2$.

	Container Type			
	I	II	III	IV
Container				
Number of containers	196	18	9	9
Inside radius, cm	3.735	7.65	6.76	6.76
Inside height, cm	11.59	21.76	25.18[a]	25.18[a]
Wall thickness, cm	0.055	0.03	0.03[a]	0.03[a]
Average weight, g	128	360	341	341
UO_2				
Height, cm	4.48[b]	19.76[c]	17.87[d]	17.87[d]
Density, g/cm^3	2.144[b]	5.505[e]	6.628	6.628
Mass, g	421	20,000	17,000	17,000
Uranium Analysis, g of U/g of UO_2	0.8754	0.8780	0.8780	0.8780
C_2H_6O - 5% H_2O in UO_2 Region				
Density, g/cm^3			0.3458	0.3458
Mass, g			711.5	711.5
$H/^{235}U$ Atom Ratio			1.556	1.556
C_2H_6O - 5% H_2O Layer Above Oxide				
Height, cm			3.14[d]	0.53[d]
Density, g/cm^3			0.8020	0.8020
Mass, g			361.5	61.5
Total Alcohol Mass, g			1,073	773

a. The thickness of the top was increased to 0.05 cm to allow for extra steel in the friction closure lid.

b. Height variations of ± 20% were measured, in 9 cans. The density is also uncertain by ± 20%. The dimensions and density result in the mass of 421 g of oxide.

c. The container was vibrated until 20,000 g filled it within 2 cm of the top prior to sealing the rolled edge lid closure. The height was not measured accurately.

d. The height of the liquid layer and the UO_2 was carefully measured at the conclusion of the experimental program 15 days after filling. The final two experiments with Type IV containers were done following removal of 300 g of alcohol.

e. The density of oxide becomes 4.999 g/cm^3 if the oxide completely fills the container. The calculations of k_{eff} described in Table 2 for arrays of this container use this density value. See discussion in text.

The average density of the UO_2 in Type I tin cans was determined to be 2.1 g/cm^3 from average depth measurements which varied from 3.6 to 5.3 cm, a variation of ±20%. However, the weighed amount in each of the 196 containers averaged 421 ± 1 g.

The Type II containers were vibrated during filling operation. The height of oxide was 2 cm below the top at the time of closure when each contained 20 kg of UO_2 and was observed to be the same when half of the containers were opened to make up the Type III fissile units. For the latter fissile units, a can was used which had a friction lid closure similar to that used on a syrup pail. The dimensions were different from the Type II so that there could be no substitution or confusion of fissile units. A weighed amount of UO_2 was added to a weighed amount of alcohol (95 wt% C_2H_6O - 5 wt% H_2O) ascertaining that the powder was wet by the liquid so that trapped air was minimal.

Upon completion of the experiments with Type III containers, they were opened for height measurements of the oxide and liquid layer on top. Settling was believed to be complete since the measurements were done 15 days after filling and 14 days after transporting to the Oak Ridge Critical Experiments Facility (CEF) from another building in the Y-12 Plant Area.

Based on a theoretical oxide density of 10.786 g/cm^3, an alcohol density of 0.8020 g/cm^3 @ 22.3°C, the measured volume of the mixture and the weights of the materials, the UO_2-alcohol mixture was 96% of theoretical, i.e., there were 4% voids in the powder not filled by alcohol. The $H/^{235}U$ atom ratio was 1.556 in the UO_2-alcohol mixture. The alcohol layer, 3.14-cm-thick in the Type III units, was reduced to 0.53 cm by removing 300 g of alcohol. This was the only difference between Type IV and Type III units.

CALCULATIONAL METHOD

The KENO Monte Carlo code with Hansen-Roach 16-group cross sections[3] was used to calculate the multiplication factor, k, of experimental arrays. This code is particularly suited for array calculations which cannot be done easily using 3-dimensional transport or diffusion codes. The results are subject to statistical errors due to the sampling process of Monte Carlo codes which are dependent upon the number of neutron histories. By using up to 5×10^4 neutron histories, the statistical error was reduced to about ± 0.004.

EXPERIMENTAL DETAILS

The experiments were performed on the Criticality Testing Unit, Horizontal Displacement,[4] formerly known as the Split Table Apparatus. It is a machine, located in the South Experimental Area of the CEF, especially designed for critical experiments.

Figure 1 is a photograph of a $7 \times 7 \times 2$ array of small cans (Type I) each containing 421 g of $U(93)O_2$. This assembly contained 1-in.-thick methyl methacrylate between fissile units. The split tables are separated a few inches and part of the reflector has been removed to show the array details. Some methyl methacrylate was used in the reflectors where sizes of polyethylene were not available. Experiments 1-4, described subsequently in Table 2, were performed with these small cans of oxide.

Figure 2 is a photograph of the $4 \times 4 \times 4$ array of 20 kg $U(93)O_2$-Type II units. The top reflector has been removed to show the close packed array. The $3 \times 3 \times 2$ arrays with and without moderator between units are shown in Figs. 3 and 4. Experiments 5-19 contained 20 kg of UO_2 in the Type II fissile units.

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3. Gordon E. Hansen and William H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1961).
4. E. R. Rohrer, W. C. Tunnell, and D. W. Magnuson, "New Critical Experiment Machines," Neutron Physics Division Annual Prog. Rept., 1961, ORNL-3193, p. 168, Oak Ridge National Laboratory (1961).

Table 2. Description of Cells and Experimental and Calculated Multiplication Factors for $U(93)O_2$ Moderated and Unmoderated Critical Experiments.

Experiment Number	Array Size	Cell Description[a] Material	Geometry	Dimensions, cm R	±X	±Y	+Z	-Z	Multiplication Factor k(exp)	k(calc)	Δk
				421 g Units of $U(93)O_2$, Type 1							
1	7 x 7 x 4	UO_2	Cylinder	3.735	--	--	+ 4.535	+ 0.055	1.0167	1.014 ± 0.005	0.003
		Void	Cylinder	3.735	--	--	+11.645	+ 0.055			
		Steel	Cylinder	3.790	--	--	+11.70	0.0			
		Void	Cuboid	--	± 4.025	± 4.025	+11.71	0.0			
		$C_5O_2H_8$	Cuboid	--	± 5.195	± 5.195	+14.05	0.0			
2	7 x 7 x 4	Exp. 1 with 18 units removed from top layer							1.0012	0.992 ± 0.005	0.009
3	7 x 7 x 4	UO_2[b]					+ 3.639	+ 0.055	--	0.988 ± 0.005	
4	7 x 7 x 4	UO_2[b]					+ 5.431	+ 0.055	--	1.056 ± 0.005	
				20 kg Units of $U(93)O_2$, Type 2							
5	4 x 4 x 1	UO_2	Cylinder	7.65	--	--	+10.88	-10.88	1.0094	1.015 ± 0.005	-0.006
		Steel	Cylinder	7.68	--	--	+10.91	-10.91			
		Void	Cuboid	--	± 7.85	± 7.85	+11.15	-11.15			
6	3 x 2 x 2	See No. 5 for Cell Description							0.9968	1.003 ± 0.005	-0.006
7	3 x 3 x 2	Void	Cuboid	--	± 9.08	± 9.08	+11.15	-11.15	1.0014	1.002 ± 0.005 0.999 ± 0.005	0.0
				20 kg Units with Methyl Methacrylate Between Top and Bottom Units							
8	3 x 3 x 2	Void	Cuboid	--	±10.22	±10.22	+11.15	-11.15	1.0001	1.005 ± 0.006	-0.005
		$C_5O_2H_8$[c]	Cuboid	--	±10.22	±10.22	+ 1.18	- 1.18		0.998 ± 0.006	
9	3 x 3 x 2	$C_5O_2H_8$[c]	Cuboid	--	±10.22	±10.22	+ 1.81	- 1.81	1.0137	1.003 ± 0.005 1.020 ± 0.006	0.011
10	3 x 3 x 2	One corner unit removed from No. 9							0.9996	0.986 ± 0.005 0.997 ± 0.005	0.008
11	3 x 3 x 2	$C_5O_2H_8$[c]	Cuboid	--	±10.22	±10.22	+ 2.41	- 2.41	1.0129	1.011 ± 0.005 1.011 ± 0.005	0.002
12	3 x 3 x 2	One corner unit removed from No. 11							0.9989	0.989 ± 0.005 1.006 ± 0.005	0.001
13	3 x 3 x 2	$C_5O_2H_8$[c]	Cuboid	--	±10.22	±10.22	+ 3.04	- 3.04	1.0049	1.004 ± 0.005 1.003 ± 0.005	0.001
				20 kg Units with 2-in.-thick Methyl Methacrylate in Three Dimensions							
14	3 x 3 x 2[d]	Void	Cuboid		±10.27	±10.27	+11.15	-11.15	--	1.066 ± 0.004[e]	
15	3 x 3 x 2[d]	Three units removed from Top Edge							1.010	1.018 ± 0.005	-0.008
16	3 x 3 x 2[d]	Three units removed from Top Edge, 2 from Opposite Corners, and 1 from the middle of an Edge							1.0018	1.017 ± 0.005	-0.015
17	3 x 3 x 2[f]	Void	Cuboid	--	±10.27	±10.27	+11.15	-11.15	1.0012	1.002 ± 0.005[g]	-0.001
		$C_5O_2H_8$	Cuboid	--	±12.70	±12.70	+11.15	-15.97			
		Void	Cuboid	--	±12.70	±12.70	+15.28	-15.97			

Table 2 (cont'd)

Experiment Number	Array Size	Cell Description[a] Material	Geometry	Dimensions, cm R	±X	±Y	+Z	-Z	Multiplication Factor k(exp)	k(calc)	Δk
18	3 x 3 x 2	Void	Cuboid	--	±10.27	±10.27	+11.15	-11.15	1.009	1.007 ± 0.005	0.002
		$C_5O_2H_8$	Cuboid	--	±12.70	±12.70	+11.15	-15.97			
		Void	Cuboid	--	±12.70	±12.70	+18.77	-15.97			
		20 kg Units with 1-in.-thick Methyl Methacrylate in 3 Dimensions									
19	3 x 3 x 2	Void[h]	Cuboid	--	±15.30	±10.22	+11.15	-11.15	1.0054	1.006 ± 0.004	-0.001
		17 kg Units with C_2H_6O-5% H_2O, Type III									
20	2 x 2 x 1	UO_2-C_2H_6O	Cylinder	6.76	--	--	+ 5.27	-12.60	1.0065	0.999 ± 0.004	0.007
		C_2H_6O	Cylinder	6.76	--	--	+ 8.41	-12.60			
		Void	Cylinder	6.76	--	--	+12.58	-12.60			
		Steel	Cylinder	6.79	--	--	+12.63	-12.63			
		Void	Cuboid	--	± 6.985	± 6.985	+12.78	-12.78			
21	2 x 2 x 1	Void	Cuboid	--	± 7.025	± 7.025	+12.78	-12.78	1.0031	1.000 ± 0.004	0.003
22	2 x 2 x 1	Void	Cuboid	--	± 7.064	± 7.064	+12.78	-12.78	1.0018	0.996 ± 0.004	0.006
23	2 x 3 x 1	Void	Cuboid	--	± 7.620	± 7.620	+12.78	-12.78	1.0146	1.019 ± 0.004	-0.004
24	2 x 3 x 1	Void	Cuboid	--	± 7.938	± 7.938	+12.78	-12.78	1.0025	0.997 ± 0.004	0.006
25	2 x 3 x 1	Void	Cuboid	--	± 8.017	± 8.017	+12.78	-12.78	0.9988	0.992 ± 0.004	0.007
26	2 x 4 x 1	Void	Cuboid	--	± 8.493	± 8.493	+12.78	-12.78	1.0003	0.997 ± 0.004	0.003
27	3 x 3 x 1	Void	Cuboid	--	± 9.049	± 9.049	+12.78	-12.78	0.9996	0.999 ± 0.004	0.001
		17 kg Units with C_2H_6O-5% H_2O, Type IV									
28	3 x 3 x 1[i]	Void	Cuboid		± 8.573	± 8.573	+12.78	-12.78	1.0042	1.000 ± 0.004	0.004
29	3 x 3 x 1[i]	Void	Cuboid		± 8.652	± 8.652	+12.78	-12.78	0.9997	1.005 ± 0.004	-0.005

a. The dimensions describe the surfaces of the geometry of the material. Each successive region surrounds the previous region but the regions may have common boundaries. Unit descriptions in subsequent experiments are omitted and the cell size depends on the dimensions of the void region.
b. The mass of UO_2 was constant; the density was changed. The cell size was unchanged.
c. Dimensions of plastic cell between top and bottom units.
d. Plastic thickness between top and bottom units was 4.82 cm and between adjacent units was 4.86 cm. The descriptions for Experiments 14 through 16 do not include this plastic.
e. When methyl methacrylate is substituted for polyethylene in 1-in.-thick layer of the reflector adjacent to the units, k(calc) = 1.078 ± 0.005.
f. Two units were removed from opposite corners of the top layer.
g. In the experiment the plastic in the cells adjacent to the reflector was polyethylene instead of methyl methacrylate. For the 2 in. plastic substitution on the bottom, a correction of -0.012 and for the 1 in. on the sides -0.008 or a total of -0.020 has been applied to the calculated k.
h. Plastic thickness between top and bottom units was 2.41 cm and between adjacent units was 2.43 cm and this plastic is not included in the description of the cells in No. 19.
i. The alcohol content for Experiments 28 and 29 was decreased from 1073 to 773 g; the 3.14 cm layer above the oxide was reduced to 0.53 cm. In the cell description the +Z dimension for the alcohol region is changed from +8.41 to +5.80 cm.

Fig. 1. Photograph of a 7 x 7 x 2 Array of 421 g Fissile Units of UO_2.

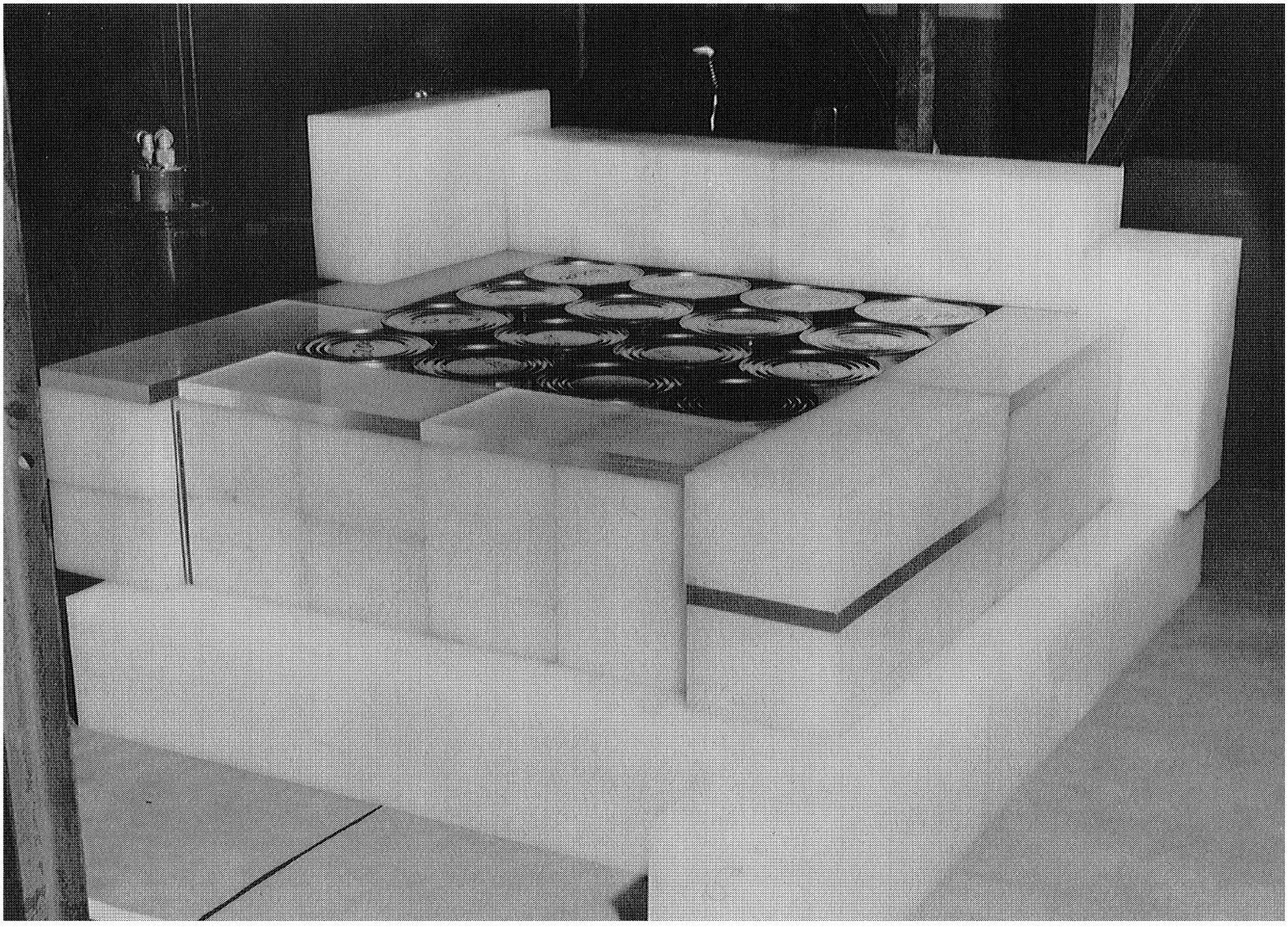

Fig. 2. Photograph of a $4 \times 4 \times 1$ Array of 20 kg Fissile Units of UO_2.

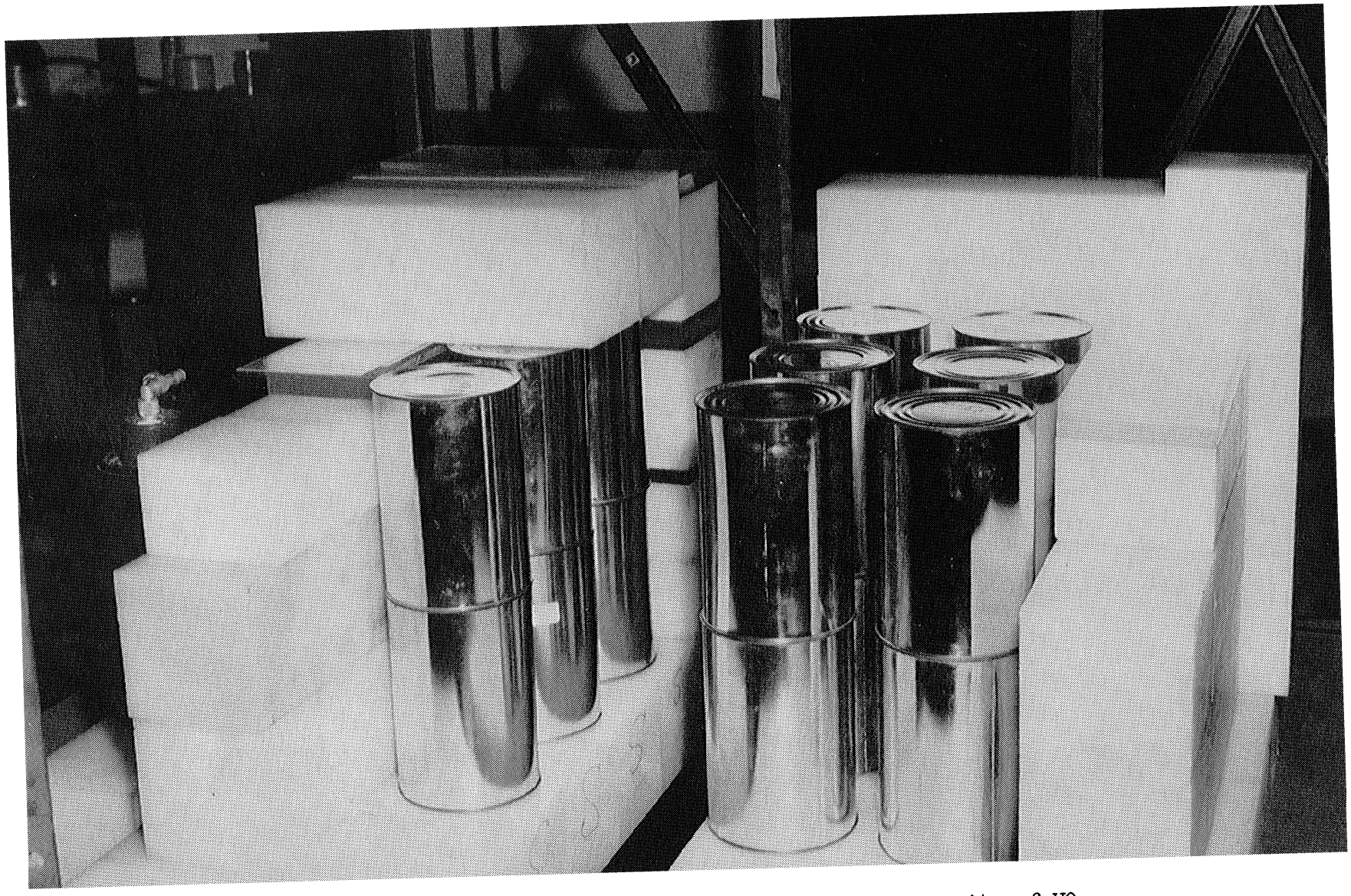

Fig. 3. Photograph of an Unmoderated 3 x 3 x 2 Array of 20 kg Fissile Units of UO_2.

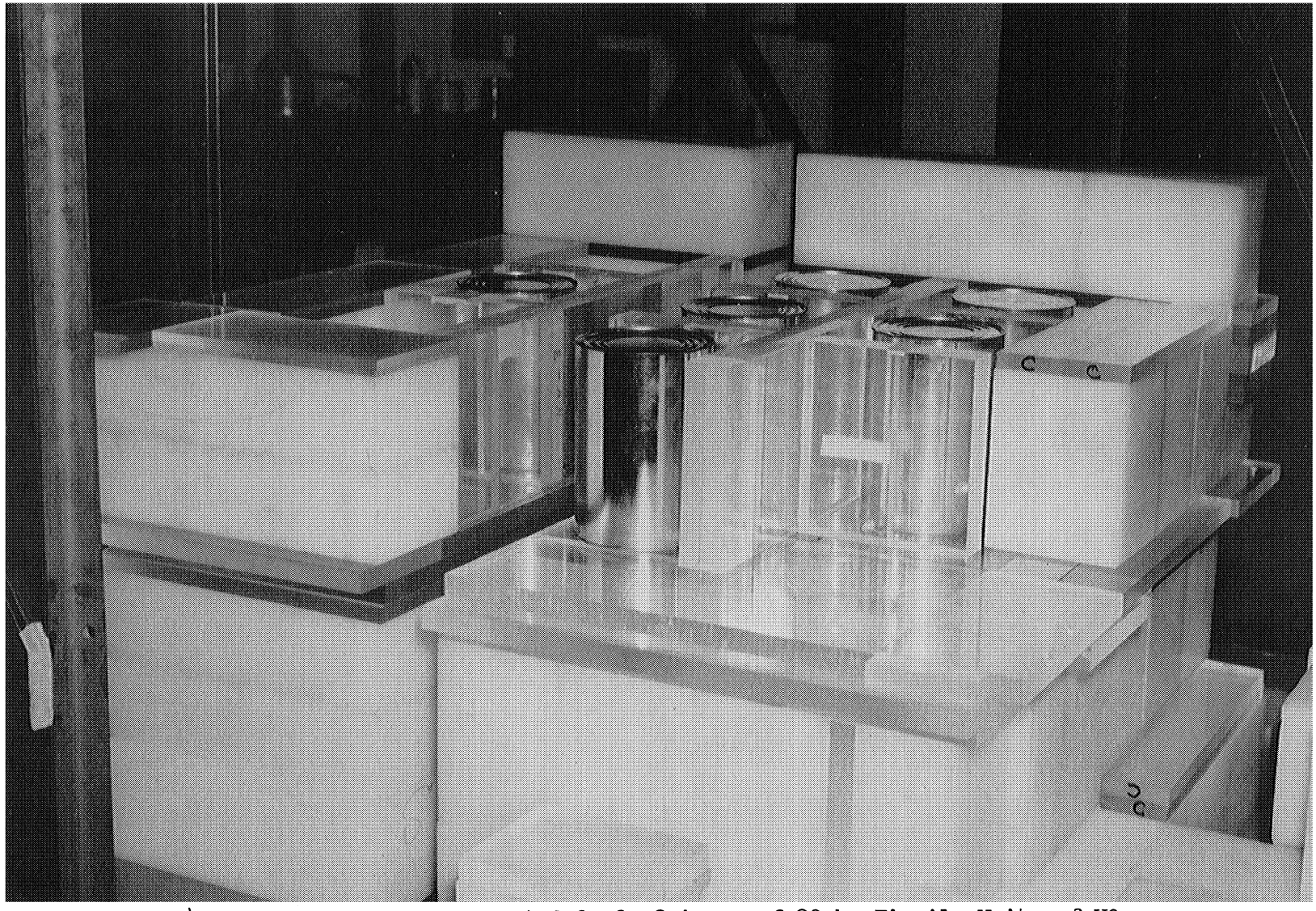

Fig. 4. Photograph of a Moderated 3 x 3 x 2 Array of 20 kg Fissile Units of UO_2 with Three Units Removed.

The 17-kg units of UO_2 (Types III and IV) contained alcohol and were used in Experiments 20-29. These containers were smaller in diameter and taller than the Type II (see Table 1 for details) and had a friction closure lid, see Fig. 5 which is a photograph of a 9-unit array. The alcohol layer thickness above the UO_2 was reduced from 3.14 to 0.53 cm for Experiments 28 and 29.

In the experimental program, criticality was sometimes achieved with the parts of the assembly separated by a few inches and the reactivity, Δk, for this gap was evaluated in various ways: sometimes units were removed; for other experiments moderator or neutron absorbers were added. Unit cell sizes (spacings of the units) were found for which there were minimal perturbations to be evaluated.

DISCUSSION OF RESULTS

The experiments with the small cans of UO_2 were planned and almost completed prior to the availability of the second batch of UO_2. Preliminary calculations for $7 \times 7 \times 4$ and $7 \times 7 \times 5$ arrays of the small 421 g units indicated that criticality could be achieved only with interspersed methyl methacrylate approximately 1 in. thick between the units. For Exps. 1 and 4, the average $\Delta k = k(exp) - k(calc) = 0.006$. In the calculational model methyl methacrylate was substituted for polyethylene moderator in the cells adjacent to the polyethylene reflector.[5] Experiments 2 and 3 describe calculations of "gedanken experiments" in which the height of the oxide in each can was varied -20% and +20%, respectively, for the variations found in the measurements. The densities of the oxide were 2.631 and 1.754 g/cm^3, respectively, for these cases. Figure 6 illustrates the dependence of the array multiplication factor on the height of oxide at constant mass in each unit. Since the calculated array multiplication factors are quite sensitive to variations of the oxide density under conditions of moderation, it was not important to evaluate other perturbations such as the plastic substitutions which were about 10 times smaller.

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5. The effect of this substitution was calculated to be less than 0.02 for the experiments with 20 kg units, see Table 2. The correction is small when compared to density effects.

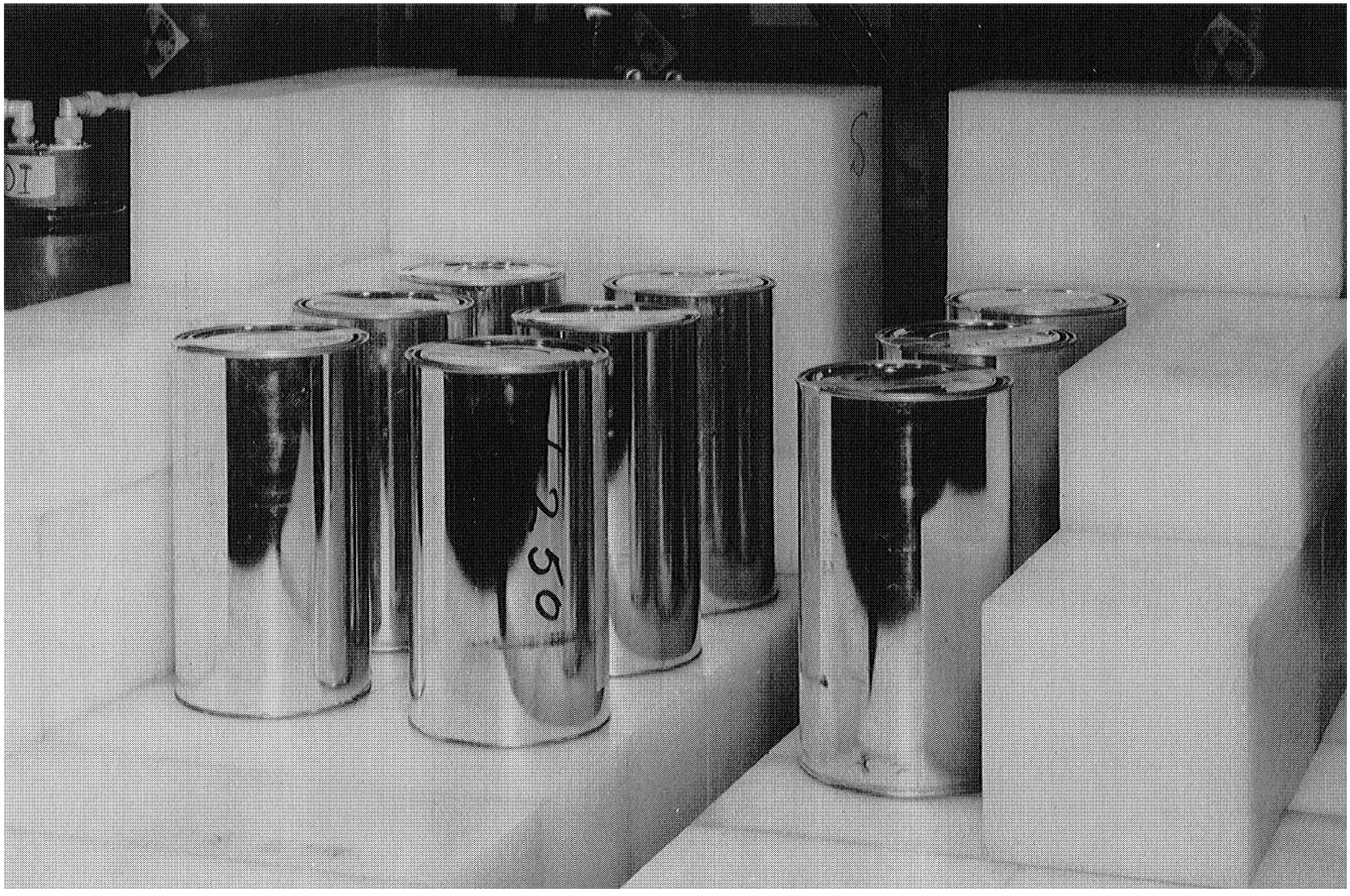

Fig. 5. Photograph of a 3 x 3 x 1 Array of 17 kg Fissile Units of UO_2-Alcohol.

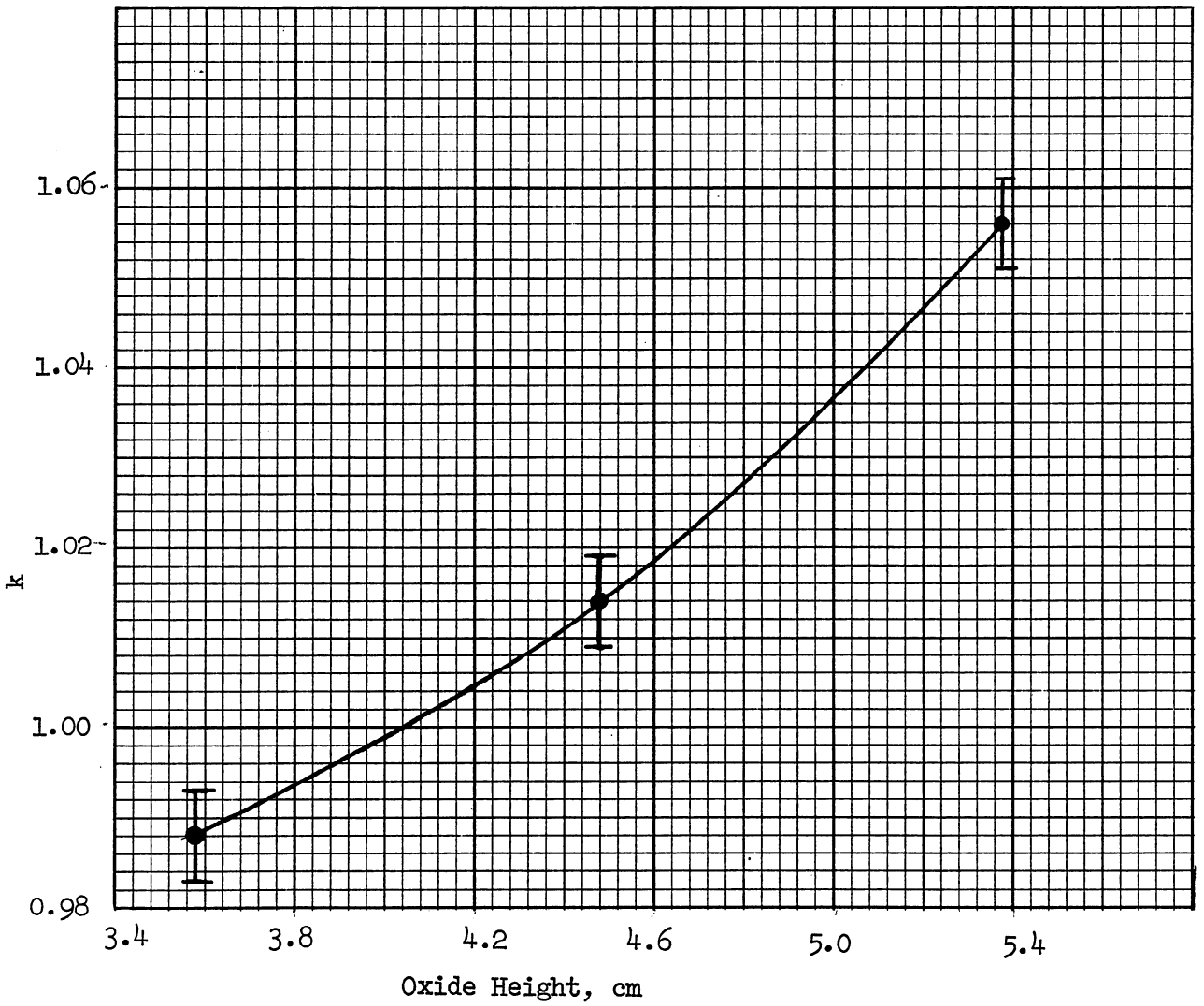

Fig. 6. Dependence of Calculated Multiplication Factor, k, on Density of Oxide in 421 g Units in a 7 x 7 x 4 Array.

The large change in reactivity due to density changes is related to the diffusion of neutrons out of the moderator and the size or surface area of the absorber or fissile material. The probability of a neutron being absorbed in the fissile material upon emerging from the interspersed moderator was dependent upon the size of the lump of fissile material. Those neutrons which were not incident on the fissile material had then a higher probability for absorption in the moderator.

The effect described above complicates the understanding of what is meant by optimum moderation in arrays because it depends upon so many different variables such as chemical form, shape, density, unit moderation, etc. What is optimum moderation for one set of conditions would not be optimum moderation for another.

The experiments with the large 20 kg units of enriched UO_2 were designed to use available materials. These were repackaged in available containers to a uniform high density by vibration, the 20 kg of uranium oxide filling each container within 2 cm of the top. The unmoderated experiments (5, 6, and 7) describe near critical systems: a 16-unit planar array, a 12-unit noncubic array, and an 18-unit air-spaced noncubic array. All arrays were reflected with polyethylene. The average $\Delta k = k(exp) - k(calc) = -0.002$ indicates that the Hansen-Roach 16-group cross sections used in the calculations of the multiplication factor in the KENO Monte Carlo code reproduce experimental k values quite well.

Experiments were performed to determine the maximum reactivity as a function of methyl methacrylate thickness between units where the plastic was inserted as a layer between the 9 top and the 9 bottom units in the 18-unit array. These are Exps. 8, 9, 11, and 13. The experiments were performed with the same spacing in the horizontal plane; the vertical separation of layers was changed only by the thickness of the plastic. Figure 7 is a plot of the multiplication factor as a function of methyl methacrylate thickness. The agreement between experimental and calculated multiplication factors is reasonably good; the average Δk, $\overline{\Delta k} = k(exp) - k(calc) = 0.005$ but one was +0.011, about 2 1/2 times the standard deviation. The observed experimental maximum in the multiplication factor is consistent with previous experience with hydrogen moderators in layers between the fissile material.

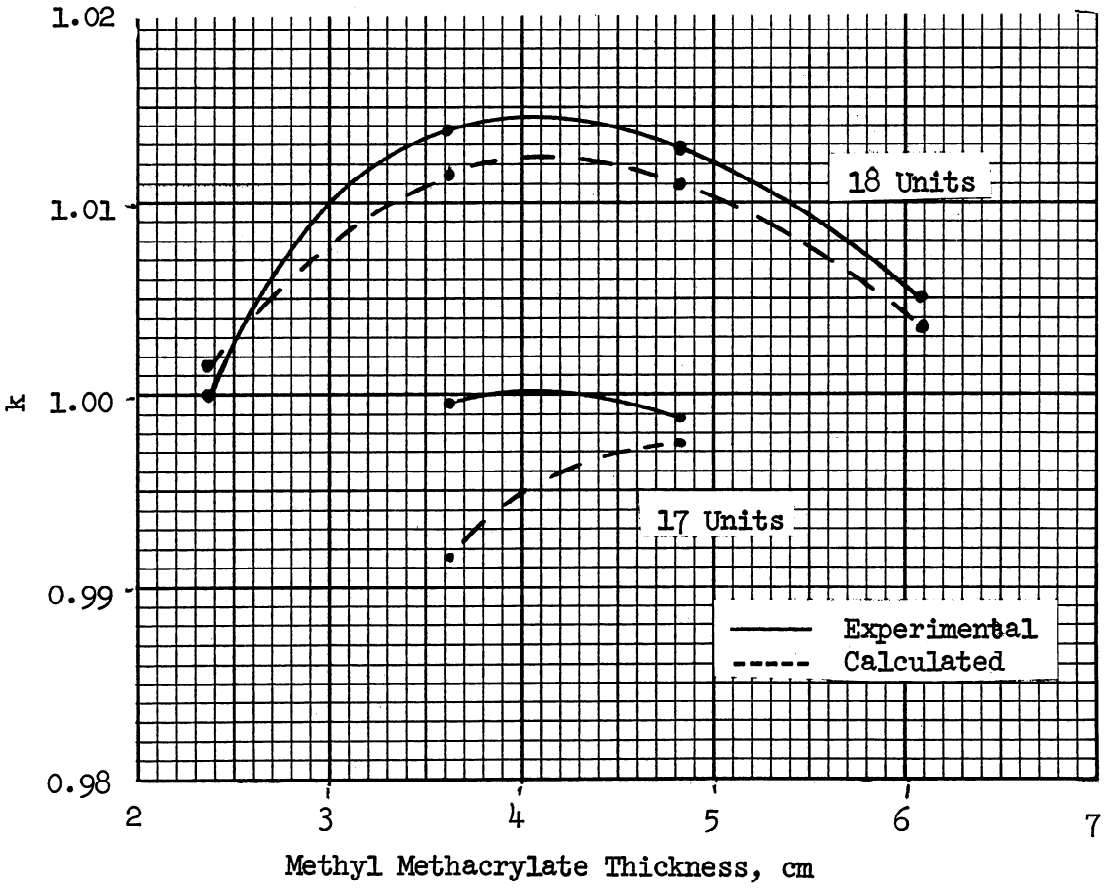

Fig. 7. Comparison of Experimental and Calculated k's for Moderated Arrays of 20 kg Units of UO_2.

The multiplication factors were recalculated for Exps. 6 and 18 assuming a 10% increase in density of the oxide; i.e., a 2-cm decrease in the height of the oxide in the units. The first experiment has no moderator between units and the latter experiment has 2-in.-thick methyl methacrylate between units. The multiplication factors of 1.004 ± 0.004 and 1.014 ± 0.004 are to be compared to 1.003 ± 0.006 and $1.015 + 0.005$, respectively. For these experiments, the 10% volume or density change has only a small effect on the multiplication factor. The order of magnitude of the statistical error is ± 0.007 when comparing the two calculations. The two experiments with only 17 units were also in reasonably good agreement, i.e., $\Delta k = 0.005$.

The experiments which illustrate the effect of "optimum moderation" with these 20-kg units are summarized by the data in Table 3. For these data the cell size of the $3 \times 3 \times 2$ (18 unit) arrays remained constant except in the vertical direction where the plastic was added so that the net reactivity change is due to the effect of the moderator and the effect of separating units. Since criticality could be achieved for two of the configurations with only 17 units (one corner unit removed from top layer) these data are also used in the comparison to calculated multiplication factors. Figure 7 is a plot of this comparison as a function of methyl methacrylate thickness for the 17 and 18 unit arrays reflected with polyethylene.

The remainder of the experiments with the 20-kg units (Exps. 14-19) were with interspersed moderator in three dimensions thereby enclosing each fissile unit effectively in a plastic box. Methyl methacrylate in two thicknesses provided the different experimental configurations. The Δk for Exps. 14-19 was -0.004.

There were two arrays in which 12 of the 18 units were displaced 1.27 cm from the center of the cells toward the reflector. There were also voids 7.62 cm high above the units in these experiments similar to the voids described in Exps. 21 and 22. For these experiments with 2.4 cm of methyl methacrylate between units, the Δk was -0.005 and -0.003, the latter with one unit of oxide removed. These are not described in Table 2.

Table 3. Polyethylene Reflected Arrays of 20 kg UO_2 Fissile Units with Various Methyl Methacrylate Thickness Between Units.

Methyl Methacrylate Layer Thickness (cm)	Multiplication Factor, k				Δk = k(exp) - k(calc)
	Experiment	1	2	Average	
		18 Units			
2.36	1.0001	1.005	0.998	1.0015	-0.0014
3.62	1.0137	1.003	1.020	1.0115	0.0022
4.82	1.0129	1.011	1.011	1.0110	0.0019
6.08	1.0049	1.004	1.003	1.0035	0.0014
		17 Units			
3.62	0.9996	0.986	0.997	0.9915	0.0081
4.82	0.9989	0.989	1.006	0.9975	0.0014
				Average	0.0023

The UO_2-alcohol mixture resulted in an $H/^{235}U$ atomic ratio of 1.556 and the internal moderation reduces the amount of fissile material required for criticality. This is exemplified by the fact that 4 of the 17 kg units could be made critical but that 16 of the 20 kg dry units were required in planar arrays. The remaining experiments, 20-29, described the air-spaced, near-critical arrays with these units. The last two experiments were done with the alcohol layer reduced from 3.14 to 0.53 cm. The average Δk for these ten experiments with alcohol-UO_2 mixtures was 0.005.

CALCULATED CRITICAL CONDITIONS

The agreement between the experimental and calculated k_{eff}'s for all of the experiments indicates that the Hansen-Roach 16-group cross sections predict criticality for undermoderated hydrogen-uranium systems. These cross sections were then used in the ANISN-S_8 transport code[6] to generate single unit critical dimensions for slab thickness, cylinder radius, and sphere radius for various water reflected UO_2-H_2O mixtures varying from 100 to 10% UO_2 or from 0 to 90% H_2O. Additional calculations were made for density variations from 50 to 100% of theoretical to determine the variation of reflected spherical critical masses as a function of density thereby determining a density exponent.

The results of these calculations are presented numerically in Table 4. Figure 8 is a plot of the critical dimensions of water-reflected infinite slabs, infinite cylinders, and spheres as a function of uranium oxide fraction for UO_2-H_2O mixtures at theoretical density. Figure 9 is a plot of the critical mass for water-reflected spheres of UO_2-H_2O mixtures having 0, 10, 20, 30, 40, and 50% void volumes. Figure 10 shows the critical masses as a function of ^{235}U density, one curve showing the variation with water content at 0% voids and the other branches illustrating the critical mass dependence upon density at

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6. Ward W. Engle, "A User's Manual for ANISN - A One-Dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," K-1693, Oak Ridge Gaseous Diffusion Plant (1967).

Table 4. Calculated Critical Parameters for Water Reflected-$U(93.2)O_2$ Mixture.

Uranium Oxide Fraction	H_2O Fraction	Water Reflected Critical Parameters: Spheres Radius (cm)	Spheres ^{235}U Mass (kg)	Spheres ^{235}U Density (g/cm^3)	Slab Thickness (cm)	Cylinder Radius (cm)	$H/^{235}U$ Atom Ratio
1.0	0.0	10.19	39.08	8.817	3.376	6.22	0.0
0.9	0.1	10.33	36.64	7.936	3.502	6.32	0.328
0.8	0.2	10.47	33.91	7.054	3.638	6.43	0.738
0.7	0.3	10.64	31.14	6.172	3.780	6.55	1.266
0.6	0.4	10.83	28.15	5.290	3.942	6.69	1.969
0.5	0.5	11.04	24.85	4.409	4.114	6.84	2.954
0.4	0.6	11.23	20.92	3.527	4.292	6.97	4.431
0.3	0.7	11.36	16.24	2.645	4.486	7.08	6.892
0.2	0.8	11.44	11.06	1.763	4.640	7.15	11.82
0.1	0.9	11.34	5.39	0.882	4.842	7.18	26.58
				10% Void Volume[a]			
0.9	0.00	11.13	45.83	7.936			
0.81	0.09	11.28	42.94	7.142			
0.72	0.18	11.43	39.71	6.348			
0.63	0.27	11.61	36.41	5.555			
0.54	0.36	11.83	33.02	4.761			
0.45	0.45	12.06	29.15	3.968			
				20% Void Volume[a]			
0.80	0.00	12.27	54.58	7.054			
0.72	0.08	12.44	51.19	6.348			
0.64	0.16	12.61	47.40	5.643			
0.56	0.24	12.81	43.48	4.938			
0.48	0.32	13.05	39.40	4.232			
0.40	0.40	13.31	34.84	3.527			
				30% Void Volume[a]			
0.70	0.00	13.71	66.62	6.172			
0.63	0.07	13.90	62.49	5.555			
0.56	0.14	14.10	57.98	4.938			
0.49	0.21	14.32	53.14	4.320			
0.42	0.28	14.59	48.18	3.703			
0.35	0.35	14.88	42.59	3.086			
				40% Void Volume[a]			
0.60	0.00	15.58	83.81	5.290			
0.54	0.06	15.79	78.52	4.761			
0.48	0.12	16.02	72.89	4.232			
0.42	0.18	16.29	67.06	3.703			
0.36	0.24	16.60	60.82	3.174			
0.30	0.30	16.94	53.86	2.645			
				50% Void Volume[a]			
0.50	0.00	18.12	109.87	4.409			
0.45	0.05	18.38	103.20	3.968			
0.40	0.10	18.65	95.83	3.527			
0.35	0.15	18.96	88.11	3.086			
0.30	0.20	19.33	80.03	2.645			
0.25	0.25	19.73	70.92	2.204			

a. Voids were added to the mixtures without changing the $H/^{235}U$ ratios given in the section of the table above.

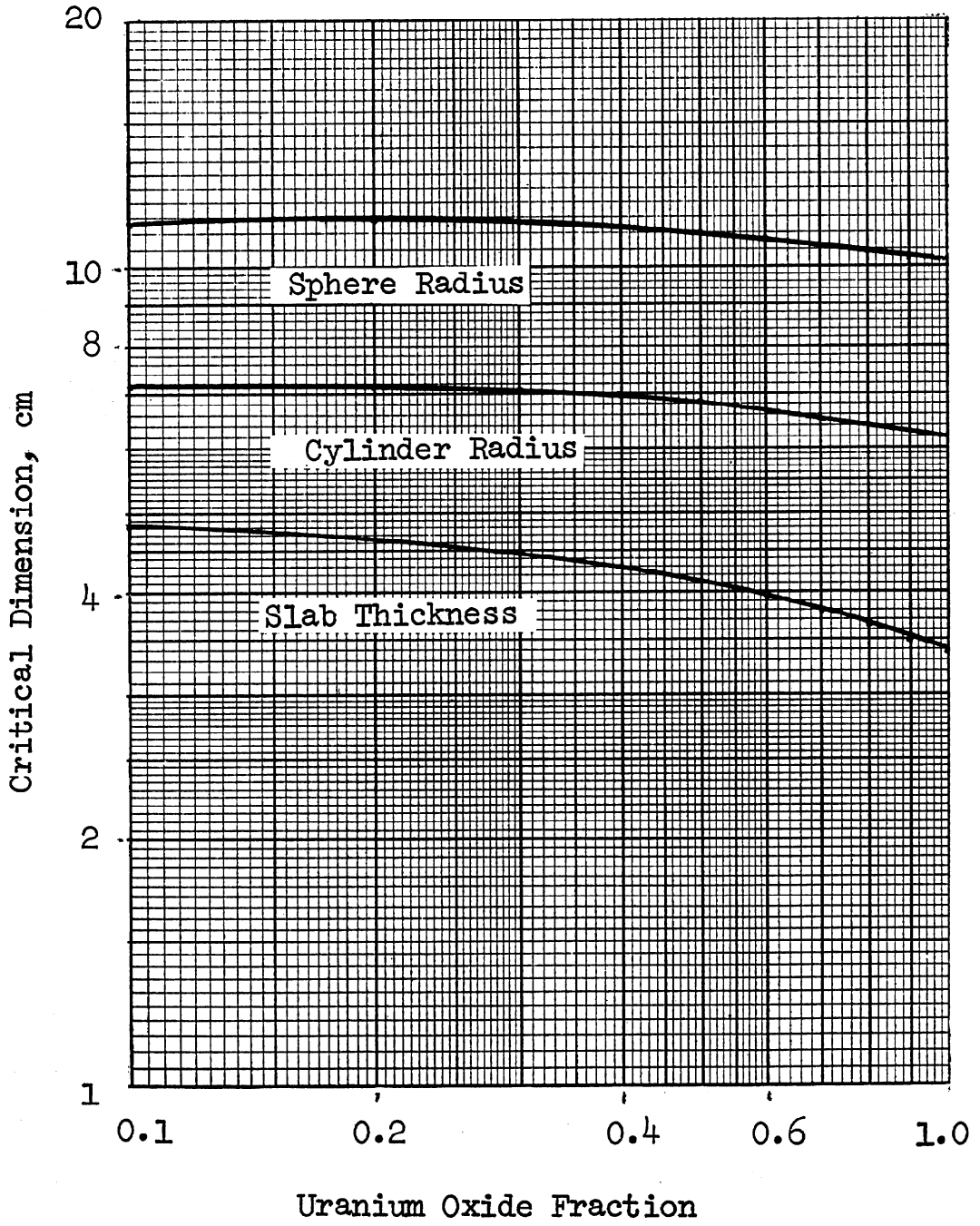

Figure 8. Calculated Critical Dimensions for Water-Reflected U(93.15)O_2-H_2O Mixtures.

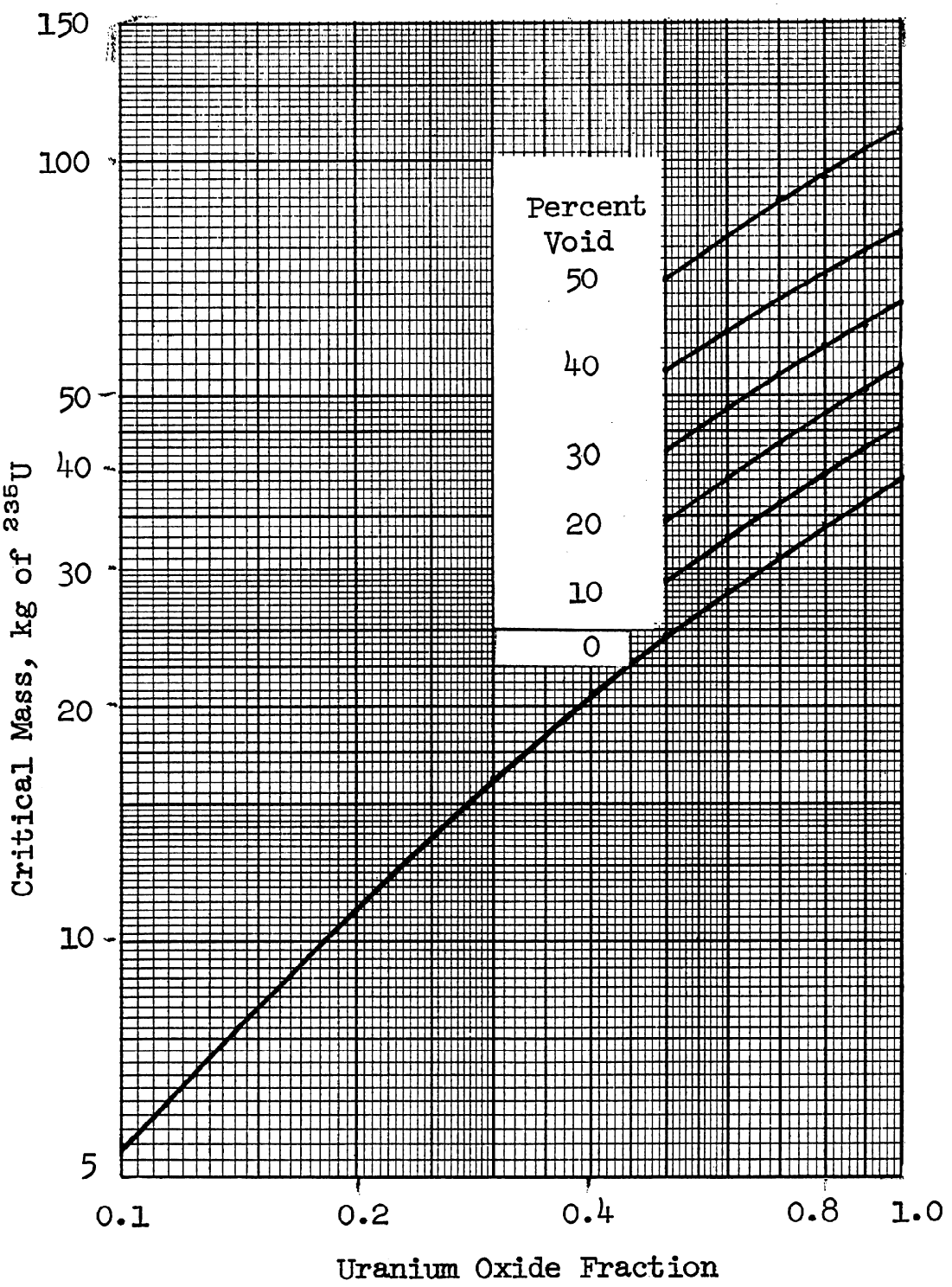

Fig. 9. Calculated Critical Masses for Water Reflected Spheres of $U(93.15)O_2$-H_2O Mixtures with Various Voids.

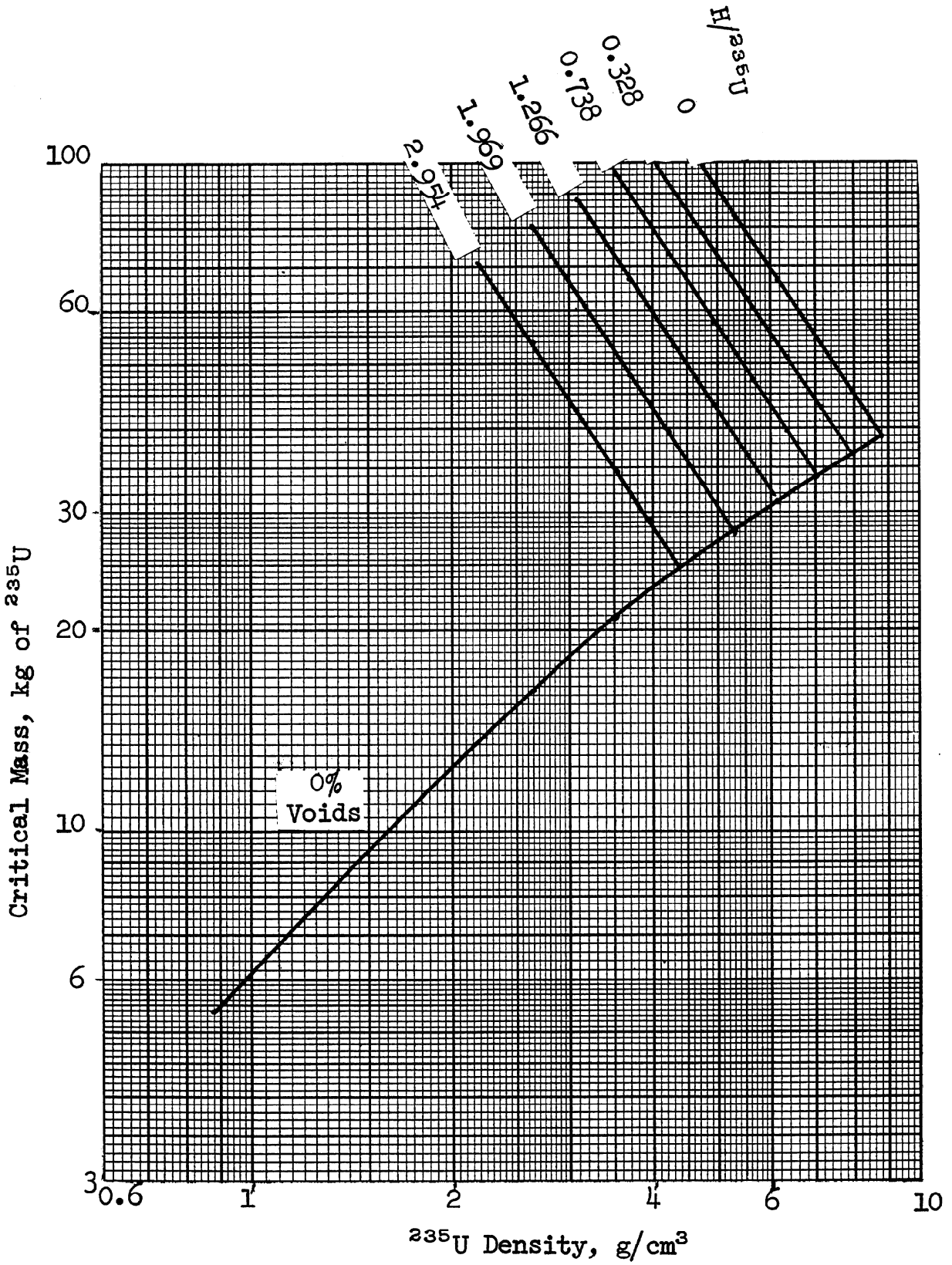

Fig. 10. Calculated Critical Masses for Water Reflected Spheres of $U(93.15)O_2$-H_2O Mixtures at Theoretical Density and for Various Voids at Constant $H/^{235}U$ Ratio.

constant $H/^{235}U$ ratio. This density relation is

$$M = M_o\left(\frac{\rho}{\rho_o}\right)^n \quad .$$

The density exponents, n, were -1.489, -1.493, -1.499, -1.502, -1.507, and -1.513 for $H/^{235}U$ ratios of 0, 0.328, 0.738, 1.266, 1.969, and 2.954, respectively, for the void fraction range of 0 to 0.5.

APPENDIX A

Analyses of UO_2 in Type II, III, and IV Containers.

Table A-1. Analyses of UO_2 in Type II, III, and IV Containers

Container Number	Uranium (wt%)	Isotopic Analysis (wt%) ^{234}U	^{235}U	^{236}U	^{238}U	H_2O (ppm)	Spectrochemical Impurity Analysis,[a] (ppm) Al	B	Co	Cr	Cu	Fe	Li	Mg	Mn	Mo	Na	Ni	Si	Sn	UO_2 Weight
6176[b]	87.840	0.99	93.14	0.21	5.66	254	22	0.2	<1	39	36	150	<0.2	40	5	28	4	74	146	18	20,005
6178[b]	87.830	1.00	93.14	0.20	5.66	286	19	0.2	1	40	40	195	<0.2	97	6	26	3	76	162	16	20,000
6180[b]	87.772	0.99	93.10	0.20	5.71	316	14	<0.1	2	40	46	184	0.3	134	5	45	3	71	159	<10	20,001
6182[b]	87.798	0.98	93.18	0.19	5.65	302	16	<0.1	<1	42	45	181	0.3	80	9	21	3	79	158	<10	20,004
6184[b]	87.781	0.98	93.18	0.19	5.65	283	17	0.4	<1	39	42	166	0.3	96	5	32	3	73	147	<10	20,005
6186[b]	87.797	0.99	93.16	0.20	5.65	329	20	<0.1	2	45	45	218	0.3	86	14	32	3	86	167	<10	19,990
6188[b]	87.803	1.00	93.15	0.30	5.55	249	18	0.3	3	47	38	194	0.3	52	6	38	2	73	180	18	19,990
6190[b]	87.765	1.00	93.14	0.37	5.49	224	18	0.2	3	54	38	233	0.5	49	9	38	2	76	198	20	20,001
6192	87.763	0.99	93.17	0.36	5.48	246	18	<0.1	1	54	33	209	0.3	27	12	18	2	73	187	<10	20,005
6194	87.806	0.99	93.17	0.36	5.48	217	18	<0.1	4	55	36	215	0.4	30	7	40	2	73	201	20	20,002
6196	87.762	0.99	93.15	0.37	5.49	181	18	<0.1	1	52	34	183	0.4	42	5	20	3	57	142	20	19,997
6198	87.802	1.00	93.15	0.45	5.40	338	17	0.2	4	83	41	278	0.5	41	9	18	6	73	202	22	20,004
6200	87.775	0.99	93.13	0.46	5.42	206	22	<0.1	3	94	43	284	0.5	33	9	17	5	74	204	25	19,999
6202	87.807	0.99	93.16	0.45	5.40	269	21	<0.1	2	91	44	320	0.5	28	9	35	6	77	211	12	20,001
6204	87.809	1.00	93.15	0.46	5.39	271	25	<0.1	<1	91	41	287	0.5	24	9	26	6	74	190	28	20,002
6206	87.818	1.00	93.14	0.47	5.39	289	21	<0.1	2	94	46	302	0.6	26	4	30	5	78	207	29	20,002
6209[b]	87.835	0.98	93.17	0.44	5.41	372	18	<0.1	<1	64	37	239	0.5	16	4	15	4	57	161	21	19,999
6211	87.824	0.98	93.17	0.38	5.47	224	20	<0.1	3	56	37	224	0.4	40	10	24	4	62	148	21	19,999
Average	87.799	0.99	93.15	0.34	5.53	270	19	<0.1	2	60	40	226	0.4	52	8	28	4	73	176	18	20,000

a. Other impurities were Au <1, Ba <2, Be <0.1, Ca <10, Cd < 0.1, Nb < 10, Pb < 8, Pd < 1, Sb < 2, Sn < 20, Ti < 4, V < 1, W < 100, and Zn < 10.
b. These containers were used to make the 9 fissile units of Types III and IV.